

The Abelian Sandpile Model and its Avalanche Polynomial

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Imagine yourself on a beach, playing in the sand. You begin to make a sandpile by adding handfuls of sand. Now you consider dropping another grain of sand onto the pile. Adding it may cause nothing to happen or it may cause the entire pile to collapse in a massive slide. This is the idea behind the Abelian sandpile model. We accomplish this task by using directed graphs where we denote one vertex as the sink and at all other vertices we have a nonnegative integer. These integers represent the number of grains of sand placed in that sandpile. When the pile gets too big an avalanche occurs sending grains along each edge adjacent to the toppling vertex. We measure the sizes of these topplings and build what is called the avalanche polynomial.

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