

Counting Sets of Mutually Orthogonal Class- r Hypercubes

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It is well known that finite fields provide a fertile ground for deriving results related to latin squares and related combinatorial objects. Using linear polynomials over finite fields, we can construct large sets of mutually orthogonal latin squares and hypercubes. Orthogonality of hypercubes, however, is much less elegant than for squares; When we speak of a pair of hypercubes of dimension $d > 2$, we must say all pairs of symbols occur equally often rather than exactly once. By extending the alphabet of latin squares to have q^r symbols rather than only q , we can define a “unique-pairs” orthogonality again if $d = 2r$.

It turns out that, in order to produce a set of mutually orthogonal class- r hypercubes, it suffices to produce a set of $r \times r$ matrices over a finite field for which each matrix has all submatrices nonsingular; and each pair of matrices has its difference nonsingular as well. It is known that the upper bound on the size of such a set is $(q - 1)^r$, though it is not known that this bound is best possible. We exhibit several examples of large sets for general r and q and see that they come close to this bound.

Matrices with no singular submatrices have been studied in the past in the slightly different context of constructing MDS codes; we will discuss the connections between the two problems and how our new construction may provide new insights.

Keywords: latin hypercubes, MDS codes, matrices over \mathbb{F}_q .