Quiz 4

1. Prove that the following two sets $A$ and $B$ are equal:

$A = \{ x \in \mathbb{Z} : x \text{ is odd} \}$ and

$B = \{ x \in \mathbb{Z} : x \text{ is the sum of two consecutive integers} \}$

Proof of $A \subseteq B$. Let $x \in A$, then there exists $a \in \mathbb{Z}$ such that $x = 2a + 1$ (because each element in $A$ is an odd integer). Since $a$, $a + 1$ are two consecutive integers, and $x = 2a + 1 = a + (a + 1)$ (i.e. $x$ is a sum of two consecutive integers) we obtain $x \in B$. Since this property is satisfied for every element in $A$, we have showed that each element in $A$ is an element in $B$. Therefore $A \subseteq B$.

Proof of $B \subseteq A$. Let $y \in B$, then there exists $b \in \mathbb{Z}$ such that $y = b + (b + 1)$, because each element in $B$ is a sum of two consecutive integers. Since $y = b + (b + 1) = 2b + 1$ is odd (by the definition of ”odd”) we have $y \in A$. Because this holds for every element in $B$, we have showed $B \subseteq A$. $\square$
2. Let $A$ be the set of all students at FAU, and let $B$ be the set of all books in the library. For each of the following sentences, write the negation of the sentence using quantifier notation. Then rewrite the negation in English.

(1) $\exists x \in A, \forall y \in B, x \text{ read } y$.

$\neg(\exists x \in A, \forall y \in B, x \text{ read } y) = \forall x \in A, \exists y \in B, x \text{ does not read } y$

Each student has not read at least one book in the library.

(2) $\forall y \in B, \exists x \in A, x \text{ read } y$.

$\neg(\forall y \in B, \exists x \in A, x \text{ read } y) = \exists y \in B, \forall x \in A, x \text{ does not read } y$

There is at least one book in the library that none of the students have read.

(3) $\exists x \in A, \exists y \in B, x \text{ read } y$.

$\neg(\exists x \in A, \exists y \in B, x \text{ read } y) = \forall x \in A, \forall y \in B, x \text{ does not read } y$

There is no student that read any book in the library.

(4) $\forall y \in B, \forall x \in A, x \text{ read } y$.

$\neg(\forall y \in B, \forall x \in A, x \text{ read } y) = \exists y \in B, \forall x \in A, x \text{ does not read } y$

There is at least one book in the library that non of the students have read.

(5) $\forall x \in A, \exists y \in B, x \text{ read } y$.

$\neg(\forall x \in A, \exists y \in B, x \text{ read } y) = \exists x \in A, \forall y \in B, x \text{ does not read } y$

There is at least one student that haven’t read any book in the library.

(6) $\exists y \in B, \forall x \in A, x \text{ read } y$.

$\neg(\exists y \in B, \forall x \in A, x \text{ read } y) = \forall y \in B, \exists x \in A, x \text{ does not read } y$

Each book in the library has not been read by at least one student.
3. Exercise for Extra Credit (5 points)

In each part of this exercise find three different sets and/or numbers $A$, $B$, and $C$ to make the statement true.

(1) $A \in B \in C$

$\begin{align*}
5 \in \{3, 4, 5, 6\} \in \{(1, 1, 0, 0), A, \triangle, \{3, 4, 5, 6\}, \Box\}.
\end{align*}$

(2) $A \subseteq B \in C$

$\begin{align*}
\{1, 2, 3\} \subseteq \mathbb{Z} \in \{1, \mathbb{Z}, A, \bigcirc\}
\end{align*}$

(3) $A \in B \subseteq C$

$\begin{align*}
3 \in \{2, 3\} \subseteq \{1, 2, 3, 4\}
\end{align*}$