Quiz 1

Consider the curve given in polar coordinates by
\[ r = 1 - \sin \theta \quad \text{(or } f(\theta) = 1 - \sin \theta). \]

a. Which of the following points (that is given in polar coordinates) is on this curve
\[ [0, 0], \ [\frac{1}{2}, \frac{\pi}{6}], \ [2, \pi]? \]

Recall: A point \( [r, \alpha] \) is on the curve defined in polar coordinates \( r = f(\theta) \), if
\[ r = f(\alpha) \]

- \([0, 0]\) is not on the curve, because \( 0 \neq f(0) = 1 - \sin 0 \)
- \( [\frac{1}{2}, \frac{\pi}{6}] \), is on the curve, because \( \frac{1}{2} = f \left( \frac{\pi}{6} \right) = 1 - \sin \frac{\pi}{6}. \)
- \([2, \pi]\) is not on the curve, because \( 2 \neq f(\pi) = 1 - \sin \pi \)

b. Find the slope of the line tangent to the curve at the point determined in a.

Recall:
\[
\left( \begin{array}{c}
\text{the slope of the line tangent} \\
\text{to the curve } r = f(\theta) \text{ at the point} \\
[f(\theta), \theta] = (f(\theta) \cos \theta, f(\theta) \sin \theta)
\end{array} \right) = \frac{f'(\theta) \sin \theta + f(\theta)(\sin \theta)'}{f'(\theta) \cos \theta + f(\theta)(\cos \theta)'}
\]

Since \( f'(\theta) = (1 - \sin \theta)' = -\cos \alpha \), and \((\sin \theta)' = \cos \theta,\)
\((\cos \theta)' = -\sin \theta\) we have
\[
\left( \begin{array}{c}
\text{the slope of the line tangent} \\
\text{to the curve } r = f(\theta) = 1 - \sin \theta \\
\text{at the point } [1 - \sin \theta, \theta]
\end{array} \right) = \frac{(1 - \sin \theta)' \sin \theta + (1 - \sin \theta) \cos \theta}{(1 - \sin \theta)' \cos \theta - (1 - \sin \theta) \sin \theta}
\]
\[= \frac{-\cos \theta \sin \theta + (1 - \sin \theta) \cos \theta}{-\cos \theta \cos \theta - (1 - \sin \theta) \sin \theta} \]
the slope of the line tangent
to the curve \( r = f(\theta) = 1 - \sin \theta \)
at the point \( \left[ \frac{1}{2}, \frac{\pi}{6} \right] \)

\[
\begin{align*}
&= \begin{pmatrix}
- \cos \frac{\pi}{6} \sin \frac{\pi}{6} + (1 - \sin \frac{\pi}{6}) \cos \frac{\pi}{6} \\
- \cos \frac{\pi}{6} \cos \frac{\pi}{6} - (1 - \sin \frac{\pi}{6}) \sin \frac{\pi}{6}
\end{pmatrix} \\
&= \begin{pmatrix}
- \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} + (1 - \frac{1}{2}) \frac{\sqrt{3}}{2} \\
- \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} - (1 - \frac{1}{2}) \frac{1}{2}
\end{pmatrix} \\
&= 0
\end{align*}
\]