Exercise

1. Rewrite the negation in English and then write these sentences using the quantifier notation.

(1) There is a smallest non-negative integer.
(2) Every integer is a product of two integers.
(3) The equation $x^2 + y^2 = 1$ has a solution in which both $x$ and $y$ are integers.
(4) Every integer can be written as a difference of two non-negative integers.

2. Let $A$ be the set of all students in our class, and let $B$ be the set of the examples considered in the lecture. Explain the meaning of the following sentences and write the negation of each of these sentences.

(1) $\exists x \in A, \forall y \in B, x$ understands $y$.
(2) $\forall y \in B, \exists x \in A, x$ understands $y$.
(3) $\forall x \in A, \exists y \in B, x$ understands $y$.
(4) $\exists y \in B, \forall x \in A, x$ understands $y$.
(5) $\exists y \in B, \exists x \in A, x$ understands $y$.
(6) $\exists x \in A, \exists y \in B, x$ understands $y$.
(7) $\forall y \in B, \forall x \in A, x$ understands $y$.
(8) $\forall x \in A, \forall y \in B, x$ understands $y$. 