Exercise

1. Write the following sentences using the quantifier notation, and label them as either True or False.

   (1) There is a smallest non-negative integer.
   (2) Every integer is a product of two integers.
   (3) The equation $x^2 + y^2 = 1$ has a solution in which both $x$ and $y$ are integers.
   (4) Every integer can be written as a difference of two non-negative integers.

2. Let $A$ be the set of all students in our class, and let $B$ be the set of the examples considered in the lecture. Explain the meaning of the following sentences.

   (1) $\exists x \in A, \forall y \in B, x$ understands $y$.
   (2) $\forall y \in B, \exists x \in A, x$ understands $y$.
   (3) $\forall x \in A, \exists y \in B, x$ understands $y$.
   (4) $\exists y \in B, \forall x \in A, x$ understands $y$.
   (5) $\exists y \in B, \exists x \in A, x$ understands $y$.
   (6) $\exists x \in A, \exists y \in B, x$ understands $y$.
   (7) $\forall y \in B, \forall x \in A, x$ understands $y$.
   (8) $\forall x \in A, \forall y \in B, x$ understands $y$. 