Modular Arithmetic

Recall: Let \( n \in \mathbb{Z} \) such that \( n > 0 \) and \( a \in \mathbb{Z}_n \).

- An element \( a \in \mathbb{Z}_n = \{0, 1, \ldots, n - 1\} \) is invertible if and only if \( \gcd(a, n) = 1 \).
- If \( \gcd(a, n) = 1 \), then there exist integers \( x, y \) such that
  \[
  1 = a \cdot x + n \cdot y
  \]
- If \( 1 = a \cdot x + n \cdot y \) and \( b = x \mod n \), then \( a \otimes b = 1 \).

1. Determine all invertible elements in
   (1) \( \mathbb{Z}_7 \).
   (2) \( \mathbb{Z}_{13} \).
   (3) \( \mathbb{Z}_{24} \).
   (4) \( \mathbb{Z}_{25} \).

2. Determine the reciprocal of any invertible element \( a \) from 1. (i.e. for each invertible element \( a \) from 1. find \( b \) in the given number system such that \( a \otimes b = 1 \).)

3. For each invertible element \( a \) from 1. find \( c \) in the given number system such that \( a \otimes c = 3 \).

4. For each not invertible element \( a \) from 1. find \( c \neq 0 \) in the given number system such that \( a \otimes c = 0 \).