1. (§4.9, #6) Find the most general antiderivative of \( f(x) = x(2 - x)^2 \).

**Solution.** Multiplying out, \( f(x) = x(4 - 4x + x^2) = 4x - 4x^2 + x^3 \). The answer is

\[
F(x) = 2x^2 - \frac{4}{3}x^3 + \frac{1}{4}x^4 + C.
\]

2. (§4.9,# 38) Find \( f \) is \( f'(x) = 4/\sqrt{1 - x^2} \), \( f(\frac{1}{2}) = 1 \).

**Solution.** The most general function with the given derivative is \( f(x) = 4 \arcsin x + C \). Setting \( x = 1/2 \), \( 1 = 4 \arcsin(1/2) + C = 2\pi/3 + C \) so \( C = 1 - 2\pi/3 \). The answer is

\[
f(x) = 4 \arcsin x + 1 - \frac{2\pi}{3}.
\]

3. (§4.9, #68) Two balls are thrown upward from the edge of the cliff in Example 7 of the text. The first is thrown with a speed of 48 ft/s and the other is thrown a second later with a speed of 24 ft/s. Do the balls ever pass each other?

(The second ball is thrown later, at less speed. It won’t go as high as the first one. Will it hit the ground before the first one, or will the first one pass it on the way down, and hit the ground first?)

The position at time \( t \) of the first ball is \( s(t) = -16t^2 + 48t + 432 \). Its velocity is \( v(t) = s'(t) = -32t + 48 \). After one second we can start the experiment again. After one second, the position of the first ball is \( s(1) = 464 \) ft and its velocity is 16 ft/s. If we now set time again to 0, we can see that the position, at time \( t \) with time zero being the moment the second ball is thrown, of the first ball is \( s_1(t) = -16t^2 + 16t + 464 \); the position of the second ball at time \( t \) is \( s_2(t) = -16t^2 + 24t + 432 \). Do these two positions ever coincide? (Before our balls hit the ground?) Equating positions:

\[
-16t^2 + 16t + 464 = -16t^2 + 24t + 432
\]

has the single solution \( t = 4 \). When \( t = 4 \), \( s_1(4) = s_2(4) = 272 \) So at 272 ft above the ground the two positions coincide. At this stage the velocities are \( s_1'(4) = -32 \times 4 + 16 = -112 \) ft/s; \( s_2'(4) = -32 \times 4 + 24 = -104 \). The first ball has a higher speed. It has overtaken the second ball and will hit the ground first. The answer to the question is **YES**.

4. (§4.9, #74) A car is traveling at 50 mi/h when brakes are fully applied producing a constant deceleration of 22 ft/s². What is the distance traveled before the car comes to a stop.

**Solution.** We need to have all units the same. Now 50 mi/h =220/3 ft/s.

Suppose the brakes are applied at time \( t = 0 \) and the car is at \( s = 0 \) at that time. Since the velocity \( v(t) \) is an antiderivative of the acceleration, we have \( v'(t) = -22 \), thus \( v(t) = -22t + C \). Setting \( t = 0 \) we get \( C = 220/3 \), so \( v(t) = -22t + 220/3 \). Now \( s(t) \) being an antiderivative of \( v(t) \), we get \( s(t) = -11t^2 + (220/3)t \) (no added constant because \( s(0) = 0 \)). The car comes to a full stop when \( v(t) = 0 \) (at which point, presumably, on stops decelerating). \( v(t) = 0 \) solves to \( t = 10/3 \). Setting this into \( s \) we get \( s(10/3) = 1100/9 \approx 56.82 \). The answer is approximately 122.22 ft.

5. (Similar to Chapter 4, Review Problem #77) A canister is dropped from a helicopter 600m above the ground. Its parachute does not open, but the canister has been designed to withstand an impact velocity of 100 m/s. Will it burst?

**Solution.** Say the canister is dropped at time 0, presumably at velocity 0. This means that its position at time \( t \) is \( s(t) = -4.9t^2 + 600 \). (Why 4.97 We are metric here, so acceleration due to gravity is 9.8 m/s².)

To find when it hits the ground, we set \( s = 0 \) and solve for \( t \), getting \( t \approx 11.01 \)s. The velocity at that time is

\[
s'(t) = -9.8t = 108.44.
\]
It is just a bit more than the canister can bear. It will burst. Probably (sometimes something designed to withstand a certain force of impact, can withstand more than it was designed for)