Confidence Interval for Proportion

Find the point estimate of the true proportion of people who wear hearing aids if, in a random sample of 676 people, 35 people had hearing aids.

\[ \hat{p} = \frac{35}{676} = 0.052 \]

36 randomly picked people were asked if they rented or owned their own home, 10 said they rented. Obtain a point estimate of the true proportion of home owners.

\[ \hat{p} = \frac{36 - 10}{36} = 0.722 \]

Find the critical value \( Z_{\alpha/2} \) that corresponds to a degree of confidence of 90%, 95%, and 99%.

You need to memorize these critical values, if you have trouble finding them by Z-score table!

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Significance Level ( \alpha )</th>
<th>Critical Value ( Z_{\alpha/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90% = 0.90</td>
<td>10% = 0.10</td>
<td>( Z_{\alpha/2} = Z_{0.05} = 1.645 )</td>
</tr>
<tr>
<td>95% = 0.95</td>
<td>5% = 0.05</td>
<td>( Z_{\alpha/2} = Z_{0.025} = 1.96 )</td>
</tr>
<tr>
<td>99% = 0.99</td>
<td>1% = 0.01</td>
<td>( Z_{\alpha/2} = Z_{0.005} = 2.575 )</td>
</tr>
</tbody>
</table>
In a survey of 300 T.V. viewers, 120 said they watch network news programs. At the confidence level 95%,

a) Find the sample proportion.
\[ \hat{p} = \frac{120}{300} = 0.4 \]

b) Find the critical value
\[ z_{\alpha/2} = z_{0.025} = 1.96 \]

c) Find the standard error
\[ se = \sqrt{\frac{\hat{p} \cdot (1 - \hat{p})}{n}} = \sqrt{\frac{0.4 \cdot (1 - 0.4)}{300}} \approx 0.0283 \]

d) Find the margin of error
\[ E = z_{\alpha/2} \cdot se = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1 - \hat{p})}{n}} = 1.96 \cdot \sqrt{\frac{0.4 \cdot (1 - 0.4)}{300}} \approx 0.055 \]

e) Find the confidence interval for the true proportion of people who watch network news programs.
\[ \text{lower bound} = \hat{p} - E = 0.4 - 0.055 = 0.345 \]
\[ \text{upper bound} = \hat{p} + E = 0.4 + 0.055 = 0.455 \]

95% confidence interval: (0.345, 0.455)

f) If we decide to decrease the margin of error to \( E = 3\% \), what sample size do I need at 95% confidence level?

Since the sample proportion is known,
\[ n = \frac{z_{\alpha/2}^2 \cdot \hat{p} \cdot (1 - \hat{p})}{E^2} = \frac{1.96^2 \cdot 0.4 \cdot (1 - 0.4)}{0.03^2} = 1024.4 \approx 1025 \]
A survey of 800 voters in one state reveals that 480 favor approval of an issue before the legislature. Construct the 99% confidence interval for the true proportion of all voters in the state who favor approval.

-- 99% confidence interval: (0.5554, 0.6446)

Of 300 items tested, 120 are found to be defective. Construct the 95% confidence interval for the proportion of all such items that are defective.

-- 95% confidence interval: (0.3446, 0.4554)

When 400 college students are randomly selected and surveyed, it is found that 280 own a car. Find a 90% confidence interval for the true proportion of all college students who own a car.

-- 90% confidence interval: (0.6623, 0.7377)

At a confidence level 90%, in order to have a margin of error of 1%, how large the sample size is required? Assume that no prior sample information is given.

Since the sample proportion is unknown,

\[
n = \frac{z_{\frac{\alpha}{2}}^2 \cdot 0.25}{E^2} = \frac{1.645^2 \cdot 0.25}{0.01^2} = 6765.06 \approx 6766
\]