Wording of Final Conclusion

- **Start**
  - Does the original claim contain the condition of equality?
    - Yes: (Original claim contains equality) → Do you reject $H_0$?
      - Yes: (Reject $H_0$) → "There is sufficient evidence to warrant rejection of the claim that . . . (original claim)."
      - No: (Fail to reject $H_0$) → "There is not sufficient evidence to warrant rejection of the claim that . . . (original claim)."
    - No: (Original claim does not contain equality and becomes $H_1$) → Do you reject $H_0$?
      - Yes: (Reject $H_0$) → "The sample data support the claim that . . . (original claim)."
      - No: (Fail to reject $H_0$) → "There is not sufficient sample evidence to support the claim that . . . (original claim)."

(This is the only case in which the original claim is rejected)
(This is the only case in which the original claim is supported)
8.3: Assumptions for Testing Claims About Population Means

1) The sample is a simple random sample.

2) The value of the population standard deviation $\sigma$ is known or unknown.

3) Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$. 
Test Statistic for Testing a Claim About a Mean

with $\sigma$ Known

$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$P$-values and Critical Values
- Found in Table A-2

with $\sigma$ Not Known

$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

$P$-values and Critical Values
- Found in Table A-3
- Degrees of freedom (df) = $n - 1$
Review: Finding P-values.

1. Start
2. What type of test?
   - Two-tailed
   - Right-tailed
   - Left-tailed
3. Is the test statistic to the right or left of center?
   - Left
     - P-value = area to the left of the test statistic
   - Right
     - P-value = twice the area to the right of the test statistic
4. P-value
5. P-value is twice this area.
6. Test statistic

Slide 4
Example 1: A sample of 106 body temperatures gives a sample mean of 98.20°F. Assume that the sample is a simple random sample and that the population standard deviation \( \sigma \) is known to be 0.62°F. Use a 0.05 significance level to test the common belief that the mean body temperature of healthy adults is equal to 98.6°F. Use the \( P \)-value method.

\[
\begin{align*}
H_0: \mu & = 98.6 \\
H_1: \mu & \neq 98.6 \\
\alpha & = 0.05 \\
x & = 98.2 \\
\sigma & = 0.62
\end{align*}
\]

\[
z = \frac{\overline{x} - \mu_x}{\frac{\sigma}{\sqrt{n}}} = \frac{98.2 - 98.6}{\frac{0.62}{\sqrt{106}}} = -6.64
\]
\[ H_0: \mu = 98.6 \]
\[ H_1: \mu \neq 98.6 \]
\[ \alpha = 0.05 \]
\[ \bar{x} = 98.2 \]
\[ \sigma = 0.62 \]

This is a two-tailed test and the test statistic is to the left of the center, so the \( P \)-value is twice the area to the left of \( z = -6.64 \). We refer to Table A1 to find the area to the left of \( z = -6.64 \) is 0.0001, so the \( P \)-value is \( 2(0.0001) = 0.0002 \).

Because the \( P \)-value of 0.0002 is less than the significance level of \( \alpha = 0.05 \), we reject the null hypothesis.

There is sufficient evidence to conclude that the mean body temperature of healthy adults differs from 98.6°F.
Example: Use the Traditional method.

\[ H_0: \mu = 98.6 \]
\[ H_1: \mu \neq 98.6 \]
\[ \alpha = 0.05 \]
\[ \bar{x} = 98.2 \]
\[ \sigma = 0.62 \]
\[ z = -6.64 \]

We now find the critical values to be \( z = -1.96 \) and \( z = 1.96 \). We would reject the null hypothesis, since the test statistic of \( z = -6.64 \) would fall in the critical region.

There is sufficient evidence to conclude that the mean body temperature of healthy adults differs from 98.6°F.
**Example:** Use the Confidence Interval method.

\[ H_0: \mu = 98.6 \]
\[ H_1: \mu \neq 98.6 \]
\[ \alpha = 0.05 \]
\[ \bar{x} = 98.2 \]
\[ \sigma = 0.62 \]

For a two-tailed hypothesis test with a 0.05 significance level, we construct a 95% confidence interval. Use the methods of Section 7.3 to construct a 95% confidence interval:

\[ 98.08 < \mu < 98.32 \]

We are 95% confident that the limits of 98.08 and 98.32 contain the true value of \( \mu \), so it appears that 98.6 cannot be the true value of \( \mu \).
Example 2:

A premed student in a statistics class is required to do a class project. She plans to collect her own sample data to test the claim that the mean body temperature is less than 98.6°F. After carefully planning a procedure for obtaining a simple random sample of 12 healthy adults, she measures their body temperatures and obtains the sample mean 98.39 and the sample standard deviation .535. Use a 0.05 significance level to test the claim these body temperatures come from a population with a mean that is less than 98.6°F.

(a) Use the Traditional method.
(b) Use p-value approach
Example 2:

\[ H_0: \mu = 98.6 \]
\[ H_1: \mu < 98.6 \]
\[ \alpha = 0.05 \]
\[ x = 98.39 \]
\[ s = 0.535 \]
\[ n = 12 \]
\[ -t_{0.05,11} = -1.796 \]

\[ t = \frac{x - \mu}{s} = \frac{98.39 - 98.6}{0.535} = -1.360 \]

(a) Use the Traditional method. Because the test statistic of \( t = -1.360 \) does not fall in the critical region, we fail to reject \( H_0 \). There is not sufficient evidence to support the claim that the sample comes from a population with a mean less than 98.6°F.

(b) \( p\text{-value} = P(t_{11} < -1.360) > P(t_{11} < -1.796) = 0.05 \), hence fail to reject \( H_0 \) at 0.05 significance level. There is not sufficient evidence to support the claim that the sample comes from a population with a mean less than 98.6°F.
Example 3: Find p-values

Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the $P$-value corresponding to the following given results.

a) In a left-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -2.007$.

b) In a right-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = 1.222$.

c) In a two-tailed hypothesis test, the sample size is $n = 12$, and the test statistic is $t = -3.456$. 
Solution:

Assuming that neither software nor a TI-83 Plus calculator is available, use Table A-3 to find a range of values for the $P$-value corresponding to the given results.

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>Area in One Tail</th>
<th>Area in Two Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.005  0.01  0.025 0.05  0.10</td>
<td>0.01  0.02  0.05  0.10  0.20</td>
</tr>
<tr>
<td>11</td>
<td>3.106  2.718  2.201  1.796  1.363</td>
<td></td>
</tr>
</tbody>
</table>
Solution (a):

a) The test is a left-tailed test with test statistic $t = -2.007$, so the $P$-value is the area to the left of $-2.007$. Because of the symmetry of the $t$ distribution, that is the same as the area to the right of $+2.007$. Any test statistic between 2.201 and 1.796 has a right-tailed $P$-value that is between 0.025 and 0.05. We conclude that $0.025 < P$-value < 0.05.
Solution (b):

b) The test is a right-tailed test with test statistic $t = 1.222$, so the $P$-value is the area to the right of 1.222. Any test statistic less than 1.363 has a right-tailed $P$-value that is greater than 0.10. We conclude that $P$-value $> 0.10$. 
Solution (c):

c) The test is a two-tailed test with test statistic $t = -3.456$. The $P$-value is twice the area to the right of $-3.456$. Any test statistic greater than 3.106 has a two-tailed $P$-value that is less than 0.01. We conclude that $P$-value < 0.01.