Section 5-3
Applications of Normal Distributions

Slide 2
Figure 5-12
Z-transformation:
Converting to Standard Normal Distribution
\[ z = \frac{x - \mu}{\sigma} \]

Slide 3
The sitting height (from seat to top of head) of drivers must be considered in the design of a new car model. Men have sitting heights that are normally distributed with a mean of 36.0 in. and a standard deviation of 1.4 in. (based on anthropometric survey data from Gordon, Clauser, et al.). Engineers have provided plans that can accommodate men with sitting heights up to 38.8 in., but taller men cannot fit. If a man is randomly selected, find the probability that he has a sitting height less than 38.8 in. Based on that result, is the current engineering design feasible?

Probability of Sitting Heights Less Than 38.8 Inches
- The sitting height (from seat to top of head) of drivers must be considered in the design of a new car model. Men have sitting heights that are normally distributed with a mean of 36.0 in. and a standard deviation of 1.4 in. (based on anthropometric survey data from Gordon, Clauser, et al.). Engineers have provided plans that can accommodate men with sitting heights up to 38.8 in., but taller men cannot fit. If a man is randomly selected, find the probability that he has a sitting height less than 38.8 in. Based on that result, is the current engineering design feasible?

Slide 4
\[ z = \frac{38.8 - 36.0}{1.4} = 2.00 \]

Probability of Sitting Heights Less Than 38.8 Inches
\[ P(x < 38.8 \text{ in.}) = P(z < 2) = 0.9772 \]

Probability of Weight between 140 pounds and 211 pounds
In the Chapter Problem, we noted that the Air Force had been using the ACES-II ejection seats designed for men weighing between 140 lb and 211 lb. Given that women’s weights are normally distributed with a mean of 143 lb and a standard deviation of 29 lb (based on data from the National Health survey), what percentage of women have weights that are within those limits?
Probability of Weight between 140 pounds and 211 pounds

\[ z = \frac{211 - 143}{29} = 2.34 \]

\[ \sigma = 29 \]

\[ \mu = 143 \]

\[ P(-0.10 < z < 2.34) = 0.99 \]

Cautions to keep in mind

1. Don’t confuse \( z \) scores and areas. \( z \) scores are distances along the horizontal scale, but areas are regions under the normal curve. Table A-2 lists \( z \) scores in the left column and across the top row, but areas are found in the body of the table.
2. Choose the correct (right/left) side of the graph.
3. A \( z \) score must be negative whenever it is located to the left half of the normal distribution.
4. Areas (or probabilities) are positive or zero values, but they are never negative.

Procedure for Finding Values Using Table A-2 and Formula 5-2

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the \( x \) value(s) being sought.
2. Use Table A-2 to find the \( z \) score corresponding to the cumulative left area bounded by \( x \). Refer to the BODY of Table A-2 to find the closest area, then identify the corresponding \( z \) score.
3. Using Formula 5-2, enter the values for \( \mu \), \( \sigma \), and the \( z \) score found in step 2, then solve for \( x \).

\[ x = \mu + (z \cdot \sigma) \] (Another form of Formula 5-2)

(If \( z \) is located to the left of the mean, be sure that it is a negative number.)
4. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.
Find $P_{98}$ for Hip Breadths of Men

$$x = \mu + (z \cdot \sigma)$$
$$x = 14.4 + (2.05 \cdot 1.0)$$
$$x = 16.45$$

The hip breadth of 16.5 in. separates the lowest 98% from the highest 2%.

The hip breadth of 16.5 in. separates the lowest 98% from the highest 2%.

Seats designed for a hip breadth up to 16.5 in. will fit 98% of men.

Finding $P_{05}$ for Grips of Women

45% 50%

Make the $z$ score negative if the value is located to the left (below) the mean. Otherwise, the $z$ score will be positive.