1. We perform an experiment of rolling a pair of fair six-sided dice.
(a) What is the probability of the event that the sum of the dice is 8?
(b) What is the probability of the event that the sum of the dice is even?

**Solution.** (a) The sample space is \( S = \{(i, j) : i, j = 1, 2, 3, 4, 5, 6\} \) with \( N(S) = 36 \). Let \( A \) be the event that the sum of the dice is 8. Then,
\[
A = \{(i, j) \in S : i + j = 8\} = \{(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)\}
\]
and \( N(A) = 5 \). So,
\[
P(A) = \frac{N(A)}{N(S)} = \frac{5}{36}.
\]
(b) Let \( B \) be the event that the sum of the dice is even. Then,
\[
B = \{(i, j) \in S : i + j = m, m = 2, 4, 6, 8, 10, 12\}
\]
\[
= \{(i, j) \in S : i, j = 1, 3, 5\} \cup \{(i, j) \in S : i, j = 2, 4, 6\}
\]
It is easy to find that \( N(B) = 18 \) (the first subset of \( B \) has \( 3 \times 3 = 9 \) pairs and the second subset also has 9 pairs but they are mutually exclusive). So,
\[
P(B) = \frac{N(B)}{N(S)} = \frac{18}{36} = \frac{1}{2}.
\]

2. If \( P(A) = 0.4, \ P(B) = 0.5, \) and \( P(A \cup B) = 0.7 \), find
(a) \( P(A \cap B) \),
(b) \( P(A' \cap B) \),
(c) \( P(A' \cup B') \).

**Solution.** (a) By the formula \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \), we find
\[
P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + 0.5 - 0.7 = 0.2.
\]
(b) Note that \( B = (A \cap B) \cup (A' \cap B) \) and the two subsets \( A \cap B \) and \( A' \cap B \) are mutually exclusive. Thus, \( P(B) = P(A \cap B) + P(A' \cap B) \), and it follows that
\[
P(A' \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3.
\]
(c) By De Morgan’s law, we have \( A' \cup B' = (A \cap B)' \). So,
\[
P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B)
\]
\[
= 1 - 0.2 = 0.8.
\]