Solutions to Quiz # 4 (STA 4032)

1. The manufacturing of semiconductor chips produces 2% defective chips. Assume that the chips are independent and that a lot contains 1000 chips.
(a) Approximate the probability that more than 25 chips are defective.
(b) Approximate the probability that between 20 and 30 chips are defective.

**Solution.** Let $X$ be the number of defective chips in the lot. Then, $X \sim b(1000, 0.02)$ binomial distribution with mean $\mu = 1000(0.02) = 20$ and variance $\sigma^2 = 1000(0.02)(0.98) = 19.6$.

(a) The desired probability can be approximated by

\[
P(X > 25) = P(X \geq 26) = P(X \geq 25.5) \approx P(Z \geq \frac{25.5 - 20}{\sqrt{19.6}}) = P(Z \geq 1.24) = 1 - P(Z < 1.24) = 1 - 0.8925 = 0.1075.
\]

(b)

\[
P(20 < X < 30) = P(21 \leq X \leq 29) = P(20.5 \leq X \leq 29.5) \approx P\left(\frac{20.5 - 20}{\sqrt{19.6}} \leq Z \leq \frac{29.5 - 20}{\sqrt{19.6}}\right) = P(0.11 \leq Z \leq 2.15) = P(Z \leq 2.15) - P(Z \leq 0.11) = 0.9842 - 0.5438 = 0.4404.
\]

2. The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of 2 hours.
(a) What is the probability that you do not receive a message during a two-hour period?
(b) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?

**Solution.** Let $X$ be the time until a message is received. Then, $X$ is an exponential random variable with $\lambda = 1/E(X) = 1/2$.

(a)

\[
P(X > 2) = 1 - P(X \leq 2) = 1 - F(2) = 1 - [1 - e^{-\lambda(2)}] = e^{-1} = 0.3679.
\]

(b) By the memoryless property, we find $P(X > 2 + 4|X > 4) = P(X > 2) = e^{-1} = 0.3679.$