Decision Support

A jump model for fads in asset prices under asymmetric information

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A R T I C L E   I N F O

Article history:
Received 14 March 2013
Accepted 15 October 2013
Available online 30 October 2013

Keywords:
Asset pricing
Asymmetric information
Fads
Instantaneous centralized moments of return
Lévy jump markets
Logarithmic utilities

A B S T R A C T

This paper addresses how asymmetric information, fads and Lévy jumps in the price of an asset affect the optimal portfolio strategies and maximum expected utilities of two distinct classes of rational investors in a financial market. We obtain the investors’ optimal portfolios and maximum expected logarithmic utilities and show that the optimal portfolio of each investor is more or less than its Merton optimal. Our approximation results suggest that jumps reduce the excess asymptotic utility of the informed investor relative to that of uninformed investor, and hence jump risk could be helpful for market efficiency as an indirect reducer of information asymmetry. Our study also suggests that investors should pay more attention to the overall variance of the asset pricing process when jumps exist in fads models. Moreover, if there are very little or too much fads, then the informed investor has no utility advantage in the long run.

1. Introduction

Asset pricing and portfolio selection problems are central issues in financial engineering. In an efficient market, it is assumed that asset prices always fully reflect available information, and all investors have the same amount of information to utilize for portfolio selection. However, one of the most striking developments of the last few decades was how the most clearly held notions of market efficiency, the positive relationship between return and non-diversifiable risk, and dividend discount models were put into question. This was due to the strong unanticipated price volatility in asset markets such as stock, bond, currency and real estate markets. Empirical studies in LeRoy and Porter (1981) and Shiller (1981) were among the first to assert that there are many market anomalies including excess volatility caused by investor overreaction and under-reaction, fads and fundamentals (mispricing). More recent behavioral finance articles such as Easley, Hvidkjaer, and O’Hara (2002), Yuan (2005), Easley, Engle, O’Hara, and Wu (2008), Bharath, Pasquariello, and Wu (2009), Caskey (2009), Blais, Bossaerts, and Spatt (2010), Hayunga and Lung (2011), Kelly and Ljungqvist (2012), Serrano-Padial (2012), Vayanos and Wang (2012) and Mendel and Shleifer (2012) also argued for the existence of these market anomalies. Consequently, it is a fact that the asset pricing and portfolio selection should be studied in an inefficient framework.

According to Kelly and Ljungqvist (2012), information asymmetry has a substantial effect on asset prices and demands which affects assets through a liquidity channel. Asset pricing models under asymmetric information rely on a noisy rational expectation equilibrium in which prices partially reveal the better informed investors’ information due to randomness in the assets supply. Examples of such studies are Grossman and Stiglitz (1980), Admati (1985), Wang (1993) and Easley and O’Hara (2004) who show that increases in information asymmetry lead to a fall in share prices and a reduction in uninformed investors’ demand for the risky asset. Thus, asymmetric information plays an important role in asset pricing models when it exists.

The link between asset mispricing and asymmetric information was first studied by Shiller (1981) and Summers (1986) in a purely deterministic and discrete setting, and later extended by Wang (1993), Guasoni (2006) and Buckley, Brown, and Marshall (2012) to the purely continuous random environment. In this framework, it is assumed that the asset has both the fundamental value and market value, and there are two types of investors: informed investors (i.e., institutional investors with internal research capabilities), who observe both fundamentals and market values, and uninformed investors (i.e., retail investors who rely on public information in order to make investment choices), who only observe market values. The difference between the market value and the fundamental value represents the current mispricing of the asset.

It is well known that asset return distributions are heavy tailed and skewed, which are at odds with the classical geometric Brownian motion models. Lévy models are among the most...
popular alternative models proposed to address this issue. Jumps in asset prices can have a very big impact on returns and mispricing. According to Summers (1986), asset prices can have large jumps away from their fundamental values. This leads to potentially large increases in fads. Consequently, the impact of fads may be more significant in affecting investment strategies and expected utility when asset prices jump. In this spirit, we propose a mispricing model under asymmetric information in a Lévy market where asset price jumps, while the mispricing is modeled by a continuous Ornstein–Uhlenbeck process and utility is logarithmic. We obtain explicit formulas for optimal portfolios and maximum expected logarithmic utilities for both the informed and uninformed investors, and prove that the optimal portfolio of each investor is more or less than its Merton optimal.

Under quadratic approximation of the portfolios, we show that the investors hold excess risky asset if and only if the ratio of first and second instantaneous centralized moments of return is greater than the Merton optimal. We also show that the excess asymptotic utility of the informed investor has an identical structure to continuous market counterpart, except that it is much less as a result of having a smaller adjusted mean reversion speed, which has a dampening effect on the excess utility. This adjusted mean-reversion speed is, a fraction of the original reversion speed of the mispricing process that has been reduced by the extra volatility arising from the jumps in the asset price. Notwithstanding the presence of asymmetric information, mispricing and jumps, our model shows that it pays to be more informed in the long run. However, if there is no mispricing, the informed investor has no utility advantage in the long run. Our study also shows that the overall variance of the asset process becomes more important for investors when jumps exist in the asset market.

The rest of the paper is organized as follows. In Section 2, we review the related literature. Section 3 presents the model, which includes filtrations and price dynamics of informed and uninformed investors. In Section 4, we consider the maximization problem of logarithmic utilities and obtain the optimal portfolios for informed and uninformed investors. Asymptotic results and quadratic approximations of logarithmic utilities and optimal portfolios are presented in Section 5. Section 6 concludes the paper. All the proofs and additional results are given in the Appendix.

2. Related literature

Discrete-time mispricing (fads) models for stocks under asymmetric information were first introduced by Shiller (1981) and Summers (1986), as plausible alternatives to the efficient market or constant expected returns assumption (cf Fama, 1970). Brunnermeier (2001) presented an extensive review of asset pricing under asymmetric information mainly in the discrete setting. He showed how information affects trading activity, and that expected return depends on the information set or filtration of the investor. These models show that past prices still carry valuable information, which can be exploited using technical (chart) analysis that uses part or all of past prices to predict future prices.

Wang (1993) presented the first continuous-time asset pricing model under asymmetric information, and obtained optimal portfolios for both the informed and uninformed investors. In this paper, investors have different information concerning the future growth rate of dividends, which satisfies a mean-reverting Ornstein–Uhlenbeck process. Informed investors know the future dividend growth rate, while the uninformed investors do not. All investors observe current dividend payments and stock prices. The growth rate of dividends determines the rate of appreciation of stock prices, and stock price changes provide signals about the future growth of dividends. Uninformed investors rationally extract information about the economy from prices, as well as dividends. Hence, in this paper, the fundamental value of the asset at any point is a function of stock price, dividend stream and dividend growth rate while mispricing is a function of dividend growth rate only.

Guasoni (2006) extends Shiller (1981) and Summers (1986) models to the purely continuous random setting. He studies a continuous-time version of these models both from the point of view of informed investor, who observe both fundamental and market values, and from that of uninformed investor, who only observe market prices. He specifies the asset price in the larger filtration of the informed investor, and then derive its decomposition in the smaller filtration of the uninformed investor using the Hitsuda representation of Gaussian processes. Uninformed investors, have a non-Markovian dynamics, which justifies the use of technical analysis in optimal trading strategies. For both types of investors, he solves the problem of maximizing of expected logarithmic utility from terminal wealth, and obtain an explicit formula for the additional logarithmic utility of informed agents. He also applies the decomposition result to the problem of testing the presence of fads from market data. An application to the NYSE-AMEX indices from the CRSP database shows that, if the fads component prevails, then the mean-reversion speed must be slow.

Buckley et al. (2012) extended Guasoni’s model for stocks following geometric Brownian motion to constant relative risk aversion investors when mispricing follows a continuous mean-reverting Ornstein–Uhlenbeck process. They obtained analogous but more general results which nests those of Guasoni (2006) as a special case of the relative risk aversion being one. Even though the notions of asymmetric information and fads in our model is analogous to Guasoni (2006), Buckley et al. (2012) and Wang (1993), we model the asset dynamics using a Lévy process motivated by Schoutens (2003), Cont and Tankov (2004), Kyprianou, Schoutens, and Wilmott (2005), Singleton (2006), Kou (2007), Øksendal and Sulem (2007) and Wu (2007). Our model applies to any asset that has a fundamental value and is simply the difference between the fundamental value and the observed value of the asset. Hence, our model is applicable to broader class of assets such as stock, bond, currency and real estate. Furthermore, we obtain the maximum expected utilities for both the informed and uninformed investors in a Lévy jump market. Hence, our model is more general and applicable, since it captures jumps, and as such, practically different from the extant literature.

It is also worth noting that our specification of the information asymmetry is different from that of the insider trading models such as Karatzas and Pirlosky (1996) and Amendinger, Imkeller, and Schweizer (1998). Like Guasoni (2006), we specify the price dynamics in the larger filtration of the informed investor, and then obtain the dynamics for the uninformed investor by contracting the larger filtration using the Hitsuda representation. In contrast, insider trading models specify the asset price dynamics of the smaller filtration of the uninformed investor. The novel information available to the insider (informed) investor is then added to the filtration of the uninformed to create the filtration of the insider investor by enlargement.

3. The model

The model consists of two assets, namely a riskless asset $B$ called bond, bank account or money market, with price $B_t = \exp \left( \int_0^t r_s \, ds \right)$, and a risky asset $S$ called asset in the sequel for simplicity. The bond earns a continuously compounded risk-free interest rate $r$, while the continuous component of asset’s percentage appreciation rate or expected return is $\mu$, at time $t \in [0,T]$. 


The asset is subject to volatility $\sigma > 0$. Market parameters $\mu_i, \tau_i$ and $\sigma_i$ are assumed to be deterministic functions.

There are two investor classes consisting of informed and uninformed investors. The informed investor indexed by $i = 1$, has knowledge of both the asset price and its fundamental or true value. Consequently, the informed investor has knowledge of the mispricing in the asset price at each time $t$, in the investment period $[0, T]$. The uninformed investor, indexed by $i = 0$, has knowledge of the asset price only. Although this investor is aware of the existence of mispricing, it cannot be observed directly. This investor therefore resorts to the use of technical analysis to assist in trading. Parameters and other characteristics of informed and uninformed investors, are indexed by “1” and “0”, respectively. Investors have filtrations (information flows) $\mathcal{H}_t^i$ contained in $\mathcal{F}$, where $\mathcal{H}_T^0 \subset \mathcal{H}_T^1 \subset \mathcal{F}$, $t \in [0, T]$. All random objects for the ith investor live on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{H}^i, \mathbb{P})$.

Although each investor observes the same asset price $S_t$, its dynamic depends on the filtration of the observer. We assume that the asset has log return dynamic:

$$d(\log S_t) = (\mu_i - \frac{1}{2} \sigma_i^2) dt + \sigma_i dW_t + dX_t, \quad t \in [0, T],$$  

where $W_t$ and $B_t$ are independent standard Brownian motions independent of $X_t$, while $U_t = (U_t)$ is a mean-reverting Ornstein–Uhlenbeck process with rate $\lambda$, and $X_t$ is a pure jump Lévy process having a sigma-finite Lévy measure $\nu$ on $\mathbb{B}(R \setminus \{0\})$, with triple $(\gamma, \nu, p)$ where $\gamma = \int_{\mathbb{R}} x^2 \nu(dx)$. $N$ is a Poisson random measure on $R_+ \times \mathbb{B}(R \setminus \{0\})$ that is linked to the asset. It counts the jumps of $X_t$ in the time interval $(0, t)$.

The return of the asset has three components; a continuous component $\mu_i^c - \frac{1}{2} \sigma_i^2$; a diffusive component $\sigma_i Y_t$, which is random, and a discontinuous component $X_t$, which is also random. The process $Y_t = (Y_t)$, the continuous random process of excess return, and the mean-reverting $\mathcal{O}$-process $U_t = (U_t)$ represent the mispricing, as defined in Guasoni (2006), where $U$ satisfies the Langevin stochastic differential Eq. (3). This admits a unique solution:

$$U_t = U_0 e^{-\lambda t} + \int_0^t \int e^{-\lambda (s-t)} dB_s = \int_0^t \int e^{-\lambda (s-t)} dB_s,$$

with $EU_t = 0$ and $EU_t^2 = \text{Var}(U_t) = \frac{1}{\lambda} (1 - e^{-\lambda t})$. Applying Itô's formula to (1) yields the percentage return dynamic:

$$dS_t = \mu_i dt + \sigma_i dY_t + \int \left( e^t - 1 \right) N(dt, dx), \quad t \in [0, T].$$  

(5)

By solving the Eq. (5), we have:

$$S_t = S_0 \exp \left( \int_0^t \left( \mu_i - \frac{1}{2} \sigma_i^2 \right) ds + \int_0^t \sigma_i dY_s + X_t \right).$$

3.1. Filtrations or information flows of the investors

For the informed investor, we take $\mathcal{H}_t^i = (\mathcal{H}_t^i)_{t \geq 0}$ to be the completed filtration generated by $W_t, B_t$ and $X_t$, augmented by the $\mathcal{P}$-null sets of $\mathcal{F}$. That is, the filtration of the informed investor is

$$\mathcal{H}_t^i \triangleq \sigma (W_t, B_t, X_s : s \leq t) \vee \sigma (N),$$

where $\mathcal{N} = \{D \subset \Omega : \exists A \in \mathcal{F}, D \subset A, \mathbb{P}(A) = 0 \}$. The uninformed observer knows only the asset price $S$ and does not have any information on the mispricing process $U$.

Although that investor may be aware that mispricing exists. These investors have filtration $\mathcal{H}_t^0 = (\mathcal{H}_t^0)_{t \geq 0}$ defined as

$$\mathcal{H}_t^0 = \mathcal{F}_t^0 \vee \sigma (X_t) \quad t \in [0, T],$$

where $\mathcal{F}_t^0 = \sigma (X_s : s \leq t)$ with $Y$ and $X$ are defined by Eqs. (2)-(4).

3.2. Asset price dynamics for the investors

We shall establish asset price dynamics for the two types of investors based on the corresponding information flows of the investors. For the informed investor, from (2) and (3), we have:

$$dY_t = -\lambda (e^{-\lambda t} - 1) N(dt, dx),$$  

where $\mu_i = \mu_i + \sigma_i \sigma_i$. For the uninformed investor, from Theorem 2.1 of Guasoni (2006), there exist an $\mathcal{F}$-Brownian motion $B_t$ and a process $v_t$ adapted to $\mathcal{F}_t$, such that for each $t \in [0, T]$, we have:

$$dY_t = v_t^2 dt + dB_t,$$

$$dS_t = \mu_i dt + \sigma_i dB_t + \int \left( e^t - 1 \right) N(dt, dx),$$

(6)

(7)

Note that $\pi_t$ is really $\pi_t(\omega)$, $\omega \in \Omega$, and hence, is a random process, $\pi_t$ is the proportion of the wealth of the informed agent that is invested in the risky asset at time $t \in [0, T]$. The remainder, $1 - \pi_t$, is invested in the bond or money market. Where it is clear, we drop the superscript “i” and simply use $\pi$ for the portfolio process.

4. Optimal portfolios and utility maximization

We obtain optimal portfolios and derive closed-form solutions of the maximum expected utilities for both the informed and uninformed investors in this section. We first need to introduce some preliminary definitions before we present our main results of this section.

4.1. Portfolios and wealth processes of the investors

**Definition 1.** The process $\pi_t = \pi_t(\omega): [0, T] \times \Omega \rightarrow R$ is called the portfolio process of the ith investor, if $\pi_t$ is $\mathcal{H}_t^i$-adapted and $E \int_0^T (\pi_t^2(r_t^i) dt < \infty$.

Note that $\pi_t$ is really $\pi_t(\omega)$, $\omega \in \Omega$, and hence, is a random process, $\pi_t$ is the proportion of the wealth of the informed agent that is invested in the risky asset at time $t \in [0, T]$. The remainder, $1 - \pi_t$, is invested in the bond or money market. Where it is clear, we drop the superscript “i” and simply use $\pi$ for the portfolio process.

**Definition 2.** The wealth process for the ith investor is $V_t^\pi = (V_t^\pi, \pi_t)$ where $V_t^\pi$ is the value of the portfolio consisting of the risky asset and bond at time $t$, when $\pi_t$ is invested in the risky asset and the initial capital is $x > 0$.

For brevity, we denote this process by $V_t^\pi = (V_t^\pi, \pi_t)$ or simply $V_t = (V_t^\pi, t \in [0, T]$ when the context is clear.
4.2. The dynamics of the wealth processes of the investors

Let $\mathcal{V}_t^i$ be the wealth of the $i$th investor at time $t$ resulting from the investment of $\pi_t^i$ in the risky asset. Assume that the bond earns continuously compounded risk-free interest rate $r$. Let $n_t = n_0$ be the number of risky assets in the portfolio at time $t$. Then $\pi_t^i = \frac{n_t}{C_0}$, where $\mathcal{V}_t^i$ is the value of the portfolio just before time $t$. It follows that

$$d\mathcal{V}_t^i = (1 - \pi_t^i)\mathcal{V}_t^i\,r_t\,dt + n_t\,dS_t = (1 - \pi_t^i)\mathcal{V}_t^i\,r_t\,dt + \pi_t^i\mathcal{V}_t^i\,\frac{dS_t}{S_t}.$$  

Therefore $\frac{d\mathcal{V}_t^i}{\mathcal{V}_t^i} = (1 - \pi_t^i)\,r_t\,dt + \pi_t^i\,\frac{dS_t}{S_t}$. By using the asset percentage return dynamics (7) and (9), the wealth process $\mathcal{V}_t^i$ of the $i$th investor has dynamic

$$d\frac{\mathcal{V}_t^i}{\mathcal{V}_t^i} = (\pi_t^i\sigma_t\,\delta_t^i + rt)\,dt + \pi_t^i\sigma_t\,dB_t^i + \int_t^T \pi_t^i(e^s - 1)\,N(ds, dx),$$  

(10)

where $\delta_t^i = \frac{\mu_t}{\sigma_t}$ is the Sharpe ratio. By applying Doleans-Dade exponentials to the Eq. (10), we find that the discounted value $\mathcal{V}_t^i = \mathcal{V}_0^i\exp\left(-\int_0^t \delta_t^i\,ds\right)$ is given by

$$\mathcal{V}_t^i = \mathcal{V}_0^i\exp\left(\int_0^t \pi_t^i\sigma_t^i\,dt + \frac{1}{2} \int_0^t \pi_t^i\sigma_t^2\,dt\right) + \int_t^T \mathcal{V}_t^i(e^s - 1)\,N(ds, dx).$$  

(11)

From the foregoing, we may effectively set the interest rate $r$ to be zero, and use the discounted wealth process $\mathcal{V}_t^i$ instead of the wealth process $\mathcal{V}_t^i$, to analyze the utility from terminal wealth. Thus $\mathcal{V}_t^i$ is equivalent to the discounted terminal wealth $\mathcal{V}_T^i$. Consequently, we maximize the utility from terminal wealth using the discounted terminal wealth $\mathcal{V}_T^i$.

4.3. Optimal portfolios and logarithmic utility maximization

For analytical tractability, assume that all investors have logarithmic utility function $u(x) = \log x$. Let $G(\pi) = \log(1 + \pi(e^x - 1))\nu(dx)$ for $\pi \in [0, 1]$. From the terminal wealth $\mathcal{V}_T^i$ given by (11) with $T = T$ and $V_0 = x$, we get

$$u(\mathcal{V}_T^i) = \log x + \int_0^T \left(\pi_t^i\sigma_t^i\delta_t^i - \frac{1}{2} \pi_t^i\sigma_t^2\delta_t^2\right)\,dt + \int_0^T \pi_t^i\sigma_t^2\,dB_t^i + \int_0^T \log(1 + \pi_t^i(e^x - 1))\,N(ds, dx).$$  

(12)

Taking expectation of (12) yields

$$\mathbb{E}[u(\mathcal{V}_T^i)] = \mathbb{E}\log \mathcal{V}_T^i = \log \mathcal{V}_0^i + \mathbb{E}\int_0^T (\pi_t^i\sigma_t^i\delta_t^i - \frac{1}{2} \pi_t^i\sigma_t^2 \delta_t^2)\,dt + \mathbb{E}\int_0^T (\pi_t^i\sigma_t^2\,dB_t^i) + \mathbb{E}\int_0^T \log(1 + \pi_t^i(e^x - 1))\,N(ds, dx).$$

Using $\log(1 + \pi_t^i(e^x - 1)) = \pi_t^i(e^x - 1) - \frac{1}{2} \pi_t^i(e^x - 1)^2 + G(\pi_t^i)$, the expected logarithmic utility from discounted terminal wealth $\mathcal{V}_T^i$ for the $i$th investor is given by

$$\mathbb{E}[u(\mathcal{V}_T^i)] = \log \mathcal{V}_0^i + \frac{1}{2} \mathbb{E}\int_0^T \left(\delta_t^i\,\pi_t^i\,\sigma_t^2\right)\,dt + \mathbb{E}\int_0^T f(\pi_t^i)\,dt,$$

(13)

where

$$f(\pi_t^i) = -\frac{1}{2} \left(\pi_t^i\sigma_t^2 - \delta_t^i\right)^2 + G(\pi_t^i).$$  

(14)

Now let $\mathcal{V}_0^i = x$. We seek a portfolio process $\pi = (\pi_t^i)_{t \geq 0}$ in an admissible set $\mathcal{A}_i(x)$ defined by

$$\mathcal{A}_i(x) = \left\{ \pi^i; \pi^i_t\text{ is } \mathcal{F}_t\text{-predictable such that } \mathcal{V}_T^i > 0 \text{ a.s. } \forall t \in [0, T] \right\}.$$  

$\pi$ is predictable if it is measurable with respect to the predictable sigma-algebra on $[0, T] \times \omega$, which is the sigma-algebra of all left continuous functions with right limits on $[0, T] \times \Omega$. The optimal portfolio for the $i$th investor is $\pi_T^i \in \mathcal{A}_i(x)$ such that

$$\mathbb{E}[\log \mathcal{V}_T^i] = \max_{\pi_T \in \mathcal{A}_i(x)} \mathbb{E}[\log \mathcal{V}_T^i].$$

That is, $\pi_T = \arg \max_{\pi_T \in \mathcal{A}_i(x)} \mathbb{E}[\log \mathcal{V}_T^i]$.

Since $\theta_t = \theta_0$ does not depend on $\pi_t^i$, we see from (13) that $\mathbb{E}[\log \mathcal{V}_T^i]$ is maximized if and only if $\mathbb{E}\int_0^T f(\pi_t^i)\,dt$ is maximized. That is, if $f(\pi)$ is maximized on the admissible set $\mathcal{A}_i(x)$, this approach is similar to the optimization method used by Liu, Longstaff, and Pan (2003), and yields the same optimal as the HJB approach. We can now present the main results of this section.

Theorem 1. Assume that $G(\cdot)$ is twice differentiable with respect to $\pi$.

(a) Let $\pi_t^i \in \mathcal{A}_i(x)$ and $h(\pi) = -\pi^2 + \frac{C_0}{\pi^2}$ with $i \in \{0, 1\}$, then $f(\pi)$ is strictly concave on $[0, 1]$ with unique maximum at $\pi^* = \frac{1}{2} = \sqrt{\frac{C_0}{E}}$, which is the optimal portfolio for the $i$th investor and satisfies $h(\pi^*) = 0$, i.e. $\pi_t^i = 1 = \sqrt{\frac{C_0}{E}}$, $t \in [0, T]$. Furthermore, if $h(0) = 0$ and $h(1) = 1$ then $h(1 - \pi_t^i + \frac{C_0}{\pi_t^i}) > 0$, there exists a unique admissible optimal portfolio $\pi_t^* = \pi_t^i$ in $\mathcal{A}_i(x)$.

(b) For the $i$th investor, the maximum expected logarithmic utility from terminal wealth is given by

$$w(x) = \max_{\pi_T \in \mathcal{A}_i(x)} \mathbb{E}[\log \mathcal{V}_T^i] = \log x + \frac{1}{2} \mathbb{E}\int_0^T (\delta_t^i)^2\,dt + \mathbb{E}\int_0^T f(\pi_t^i)\,dt,$$

where $\pi_t^* = \pi_t^i$ in $\mathcal{A}_i(x)$ is the optimal portfolio for the $i$th investor.

We first note that the optimal portfolio $\pi^* = \pi_t^i$ is random in general and becomes deterministic when there is no information asymmetry. When there is information asymmetry, the drift term in the return dynamics is random, and hence, the optimal demand for the risky asset is random. If there is no information asymmetry, then both investors observe both fundamental and market values of the asset. Hence, there is no mispricing, which means $U \equiv 0$, and therefore the return dynamics are the same, yielding a common deterministic Merton optimal for the demand of the risky asset.

We can also analyze our results in Theorem 1 in terms of continuous and discrete parts, in line with the Merton’s GBM model. The Merton (1971) optimal portfolio $\pi_{\text{M}}^i$ for an asset with GBM dynamics: $dS_t = \mu_t\,dt + \sigma_t\,dB_t$, is $\pi_{\text{M}}^i(t) = \frac{\mu_t - r_t}{\sigma_t^2}$, where $\pi_{\text{M}}^i$ is the Merton optimal. The Merton optimal $\pi_{\text{M}}^i$ is the excess asset holding resulting from the jumps for the $i$th investor. Furthermore, part (b) establishes

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1 The classical definition of the Merton (1971) optimal portfolio, $\pi_{\text{M}}^i$, is presented for an asset with GBM dynamics, and $\pi_{\text{M}}^i$ is deterministic. In our case, $\pi_{\text{M}}^i$ is defined for asset dynamics $dS_t = \mu_t\,dt + \sigma_t\,dB_t$, for each investor with mispricing, and $\pi_{\text{M}}^i$ is random. However, we still call $\pi_{\text{M}}^i$ the Merton optimal portfolio as the underlying definitions are identical.
that the maximum expected logarithmic utility from terminal wealth
for each investor, having \( x > 0 \) in initial wealth, is given by

\[
G'((\pi)) = -\int_0^{\pi} \left( e^{x-1} - 1 \right) v(dx) < \infty.
\]

This is required to ensure that the second derivative of the partial objective function \( G((\pi)) \) exists which then leads to the existence of an optimal portfo-
lio. If the Lévy measure is such that the second derivative exists in a
wider interval \([a,b]\), with \( a < 0 \) and \( b > 1 \), then the restriction of
no short selling and no borrowing is not required. Alternatively, when
\( G((\pi)) \) is defined as an integral dependent on the Lévy measure, one
could numerically determine its support, and hence the admissible
set, by finding the largest interval \([a,b]\) for which the integral and
its derivatives exist. If \( a < 0 \) and \( b > 0 \), let \( \pi < 0 \) and \( \pi > 1 \)
respectively. This restriction holds for most other cases such as markets
driven by Variance Gamma (cf Madan & Mil-
nan, 1991; Madan, Carr, & Chang, 1998; Madan & Seneta, 1990 &
Barndorff-Nielsen, 1995), CGMY (cf Carr, Geman, Madan, & Yor,
2002), and Jump-diffusion processes (cf Merton, 1971; Kou, 2002).

4.4. Excess asset holdings of investors

The following useful result is presented without the superscript
“i”.

**Proposition 1.** Assume that \( \pi \in [0,1] \) and \( \int_0^\pi \left( e^{x-1} - 1 \right) v(dx) < \infty \).

Let \( \pi = \pi_2^\infty \) be the Merton optimal for each investor. Then, there
exists \( \phi_\infty \) between \( \pi \) and \( \pi_2^\infty \) such that the optimal portfolio
that the maximum expected logarithmic utility from terminal wealth

\[
\pi^* = \pi + \frac{G((\pi))}{G'((\pi))}.
\]

For each investor, the excess asset holding over its Merton optimal
\( \pi \), is \( \pi_2^\infty - G((\pi)) \). Note that this excess holding of asset is
strictly due to the presence of jumps. Observe that we have a posi-
tive or negative excess over the Merton optimal in lock-step with
the sign of \( G((\pi)) \). That is, the investor holds a positive or negative excess depending on whether \( G \) is increasing or decreasing at the
Merton optimal, \( \pi_1 \).

In general, we have to use numerical methods to compute \( G((\pi)) \) as
it may not be possible to find the value analytically. However,
the numerical procedure should be viable for most of the widely
used jump processes. Here we provide some sufficient conditions
on the sign of \( G((\pi)) \) for Kou (2002) Jump-Diffusion model. In the
Kou model, jump sizes follow an asymmetric double-exponential
distribution with Lévy density of the form

\[
v(dx) = \kappa_1 p_{\kappa_1} e^{-\kappa_1 x I_{(x > 0)}} + (1 - p_{\kappa_1}) e^{-\kappa_1 x I_{(x < 0)}} dx,
\]

where \( \kappa_1 > 0 \). \( \kappa_1 > 0 \) govern the decay of the tails for the distribu-
tion of positive and negative jump sizes respectively and \( p_{\kappa_1} \in [0,1] \)
represents the probability of an upward jump. Here \( \kappa_1 \) is the

intensity of the Poisson process that drives the jumps-diffusion pro-
cess. After some basic calculation, we find that

\[
G((\pi)) = \kappa \int_0^\infty \left[ p_{\kappa_1} e^{-\kappa_1 x} \left( (2e^x - 1)^2 \right) \right] (1 - p_{\kappa_1}) e^{-\kappa_1 x} \right] \frac{(1 - p_{\kappa_1}) e^{-\kappa_1 x} \right]}{(1 - x)^2} dx.
\]

Now let \( \rho = \sqrt{\kappa_1} \). Then, in terms of the values of the
involved parameters, some sufficient conditions on the sign of \( G((\pi)) \) can be
categorized as follows. First, in the case of \( \kappa_1 = \kappa_2 \), we have that
\( (a) \) if \( p_{\kappa_1} > \frac{1}{2} \) and \( \kappa_2 < \frac{1}{\sqrt{\kappa_1}} \), then \( G((\pi)) > 0 \); \( (b) \) if \( p_{\kappa_1} < \frac{1}{2} \) and \( \kappa_2 > \frac{1}{\sqrt{\kappa_1}} \),

\( G((\pi)) > 0 \). Second, in the case of \( \kappa_1 > \kappa_2 \), if \( p_{\kappa_1} = \frac{1}{2} \) and \( \kappa_2 \in \left( \frac{1}{2}, \frac{1}{\sqrt{\kappa_1}} \right) \), then \( G((\pi)) < 0 \). Third, in the case of \( \kappa_1 < \kappa_2 \), if \( p_{\kappa_1} = \frac{1}{2} \) and \( \kappa_2 \leq \frac{1}{\sqrt{\kappa_1}} \), then \( G((\pi)) > 0 \). This type of character-
ization using the model parameters would be useful for an investor
to determine whether he has to hold an excess in practice.

5. Asymptotic utilities and their quadratic approximations

We present the asymptotic results of the maximum expected utility
and approximated results of the utilities and optimal portfo-
lios of both the investors in this section. These results are impor-
tant to understand the long run behavior of the investors. We set
\( r = 0 \) without loss of generality.

5.1. Asymptotic utilities of investors

Let \( u_i((\pi)) \) be the maximum expected logarithmic utility of the
ith investor resulting from an optimal portfolio \( \pi^{-1} \) over the investment
horizon \( T \). Since \( r = 0 \), the asset’s Sharpe ratio for the ith investor
is \( \theta = \frac{\mu}{\sigma} = \frac{\mu}{\sigma} + \frac{\pi^{-1}}{\sigma} \). We now provide the asymptotic
utilities of investors.

**Theorem 2.** Let \( x > 0 \) be the initial capital of the investors and
\( i \in [0,1] \). As \( T \rightarrow \infty \), the maximum expected logarithmic utility from
terminal wealth for the ith investor, is \( u_i^T((\pi)) \) given by

\[
u_i^T((\pi)) = \frac{\pi_i^{-1}}{\sigma_i} \int_0^T \frac{\mu_i}{\sigma_i} \left( \frac{\pi_i^{-1}}{\sigma_i} \right)^2 dt + \frac{\pi_i^{-1}}{T} \left( 1 - p_i \right) \left( (1 + (-1)^{\pi_i^{-1}}) \right) \frac{\sigma_i}{\pi_i}
\]

where \( p_i \in [0,1] \) and \( u_i^T((\pi)) \) is defined as the utility for the ith investor
and \( \phi_i((\pi)) \) is defined as the utility for the informed investor.

Moreover, the asymptotic excess utility of the informed investor
is

\[
u_i^T((\pi)) - \frac{\pi_i^{-1}}{T} \left( 1 - p_i \right) \left( (1 + (-1)^{\pi_i^{-1}}) \right) \frac{\sigma_i}{\pi_i} \phi_i((\pi))^{-1} \left( \phi_i((\pi))^{-1} \right).
\]

**Theorem 2** provides the long-term behavior of the logarithmic
utilities for both informed and uninformed investors. Compared
to the purely continuous fads models, we have extra asymptotic
utilities \( u_i^T((\pi)) \) due to jumps, which depend on the equilibrium
states of the optimal portfolios. In addition, the asymptotic excess
utility of informed investor also has an extra term due to jumps.
However, it is not easy to give analytic expression for this extra
excess utility. In the Appendix, we give an approximated explicit
formula for \( \phi_i((\pi)) \) based on our results in next sub-section.

5.2. Optimal portfolios and utilities under quadratic approximation

In this section, for \( x = 0 \) (Merton optimal portfolio), we derive
some useful formulas based on the assumption that \( G((\pi)) \) is
approximated by a Taylor expansion built from two instantaneous
centralized moments \( M_1 \) and \( M_2 \), defined below. This approxima-
tion leads to very nice consequences under the assumption that
there exists \( k > 2 \), such that \( \int_0^\infty \left( e^{x-1} - 1 \right) v(dx) < \infty \) to
ensure that \( G((\pi)) \) exists on \( [0,1] \) and \( G'((\pi)) < 0 \).
We first introduce some extremely useful objects linked to the Lévy measure that will be instrumental in computing approximations.

**Definition 3.** (Instantaneous Centralized Moments of Return) Let \( j \in \{1, 2, \ldots, k\} \). Define the \( j \)th instantaneous centralized moment of return for the asset with dynamics (1) by the prescription:

\[
M_j \triangleq \int_0^1 \nu(x) \, dx.
\]

Here, \( M_j \) is well defined because \( \nu(x) < \infty \), and the quadratic variation is

\[
\mathbb{C}^Q(0) = \left( (j-1)! \right) \int_0^1 \nu(x) \, dx = (-1)^{j-1} (j-1)! M_j .
\]

Now define the functions \( A, B, C \) of \( t, \sigma, M_1, M_2 \), by the prescriptions:

\[
A_i \triangleq \frac{-M_2}{2(\sigma_i^2 + M_2)}; \quad B_i \triangleq \frac{M_1 \sigma_i}{(\sigma_i^2 + M_2)}; \quad C_i \triangleq \frac{M_1^2}{2(\sigma_i^2 + M_2)}.
\]

Let \( Q(\theta) = Q(\theta : \sigma_1, M_1, M_2) = A \theta^2 + B \theta + C \), \( A_i = \lim_{t \to \infty} A_i \), \( \sigma_i = \lim_{t \to \infty} \sigma_i \). We have the following important result.

**Theorem 3.** (a) Let \( G(x) \) be defined on \([0, 1]\). Under quadratic approximation of \( G \), the optimal portfolio for each investor is

\[
\pi' \approx \frac{M_1 - M_2 x}{ \sigma^2 + M_2} .
\]

(b) The jump component of the maximum expected utility for the \( i \)th investor resulting from quadratic approximation of \( G \) is

\[
u_i u_i(x) \approx \mathbb{E} \int_0^T \left( \int_0^x Q(\theta : \sigma_i, M_1, M_2) \, d\theta \right) \, dt, \quad i \in \{0, 1\}.
\]

(c) Under quadratic approximation, as \( T \to \infty \), the maximum expected logarithmic utility from terminal wealth for the \( i \)th investor with \( x > 0 \) in initial wealth, is

\[
\nu_i u_i(x) \sim \log x + \frac{1}{2} \int_0^x \left( \frac{\mu_i}{\sigma_i} \right)^2 \, dt + \int_0^x Q(\theta : \sigma_i) \, d\theta + \frac{1}{2} (1 - p)(1 - (1 - p) T).
\]

We first observe from part (a) of Theorem 3 that under quadratic approximation, an investor holds excess risky asset if and only if the ratio of first and second instantaneous centralized moment of return \( \frac{M_1}{M_2} \) is greater than the Merton optimal, \( z \). These moments are determined strictly by the Lévy measure driving the jumps in the asset price and can be computed easily for a given Lévy density.

We also note that part (d) of Theorem 3 is analogous to Guasoni (2006)'s major result of Theorem 3.1 for excess asymptotic utility, given by \( \frac{1}{2} (1 - p)(1 - p) T \), where \( \lambda \) is the mean-reversion rate for the Ornstein–Uhlenbeck process. Its maximum is still achieved at \( p = \frac{1}{2} \), which is equivalent to a market where the mispricing level is at 75%. Note in our case, the influence of the mean-reversion rate for the O–U process on the expected logarithmic utilities is reduced to \( \varpi = \lambda \frac{\sigma^2 + M_2}{\lambda^2 M_2} \), instead of \( \lambda \) as in the purely continuous GBM market. The quantity \( \sigma^2 + M_2 \) is the total variance or squared volatility of the asset pricing process, where \( \lambda \) is the contribution due to jump risk. Thus, the effective mean reversion speed of the mispricing process is reduced to a fraction of its original speed when jumps exist, thereby reducing the overall long-run excess utility advantage of the informed investor in contrast to the strictly GBM continuous case where asset price diffuses only without jumps. Consequently, if mispricing is continuous, jumps in asset price appear to add more efficiency to the market by reducing the end of period utility advantage of the informed investor. That is, it appears that jumps indirectly increase the informational advantage of the uninformed investor (information asymmetry) by directly reducing the utility advantage of the informed investor. This is a surprising consequence of our model. However, if the diffusive variance \( \sigma^2 \) is large relative to the jump variance \( M_2 \), then for all practical purposes, the excess utility is essentially its continuous counterpart, which is its maximum possible level.

![Fig. 1. Excess utility versus proportion of fads (\( q^2 \)) and volatility (\( \sigma \)).](image-url)
It is also worth noting that, whether or not jumps exist, there is no excess utility advantage of the informed investor when the market is totally free of mispricing or is completely mispriced. The excess expected optimal utility is proportional to \( p(1 - p) \), where \( p = \sqrt{1 - q^2} \) and \( q^2 \) is the proportion of mispricing. If there is no mispricing, then \( q^2 = 0 \) and the fundamental and asset prices are equal. This yields \( p = 1 \), and hence there is no excess utility. If the market is completely mispriced, then \( q^2 = 1 \), and hence \( p = 0 \) which leads to no excess utility. Equivalently, both return dynamics are equal since they are driven by a common Brownian motion \( B \). This yields the same optimal demand for the risky asset for each investor, and hence the same expected optimal utility. Thus, there is no difference in optimal utility. Therefore, superior knowledge of the informed investor does not translate to any utility advantages when the market is symmetric \( (q = 0) \) or totally asymmetric \( (q = 1) \). However, from a psychological standpoint, there is comfort in being more informed, even if excess utility is non-existent.

We plot the surface of the excess utility over the proportion of mispricing, \( q^2 \) and the volatility, \( \sigma \) for the Kout (2002) Jump-Diffusion model with Lévy density given by Eq. (15). We use \( \kappa = 10 \) per year, \( 1/\kappa = 2\% \), \( 1/\kappa = 4\% \), \( p_0 = 0.3 \), \( T = 1 \) year in Fig. 1. The computed value of \( M_2 = 0.0225 \) in this case. Observe that we have maximum excess utility when mispricing is at \( 75\% \) (i.e. \( q^2 = 0.75 \)). Excess utility increases at a decreasing rate with mispricing up to \( 75\% \) and decreases thereafter at faster rates. It also increases with the diffusive component of the overall variance of the asset process when jump exists. However, if there are no jumps in asset price (i.e. \( M_2 = 0 \)), then the excess utility is surprisingly independent of the overall variance of the asset price process. Thus, our results also show that the overall variance of the asset price process becomes more important for investors when jumps exist in the asset market.

6. Conclusions

We studied how asymmetric information, fads and Lévy jumps in the price of an asset affect the optimal portfolio strategies and maximum expected utilities of informed and uninformed investors in a financial asset market, and derived some explicit and informative results about the optimal portfolios and expected logarithmic utilities of each investor. Moreover, our study analytically enhances the extant literature on the fads models for asset under asymmetric information.

Under quadratic approximation of the portfolios, we show that excess assets are held by an investor if and only if the ratio of the first and second instantaneous centralized moments of return is greater than the Merton optimal portfolio of that investor. We also show that jumps reduce the excess long run utility for the informed investor relative to that of the uninformed investor, which implies that jump risk may be good for market efficiency as an indirect reducer of information asymmetry. In the presence of asymmetric information, mispricing and jumps, our model shows that it pays to be more informed in the long run, but the excess utility of the informed investor is less than that obtained in the purely continuous GBM market. However, if there is no mispricing (or too much mispricing), the informed investor has no utility advantage. Our results also suggest that investors should pay more attention to the overall variance of the asset process when jumps exist in the financial asset market.

Acknowledgements

The authors thank Lorenzo Peccati and anonymous reviewers for the prompt review process and their helpful comments. Winston Buckley also gratefully acknowledges financial support received from a Dorothy Graham Fellowship.

Appendix: Proofs and additional results

Proof of Theorem 1

(a) For simplicity, we drop the superscript \( i \) in the proof. Let \( f = f^i \) and assume that \( G^i(\pi) \) exists on \( \mathcal{R} \). From (14), \( f^i(\pi) = -\frac{1}{2} \sigma_i (\pi \sigma_i - \theta_i)^2 + G^i(\pi) \), whence \( f^i(\pi) = -\sigma_i (\pi \sigma_i - \theta_i) + \frac{1}{2} \sigma_i (\pi \sigma_i + \theta_i) \) and \( f^{i+1}(\pi) = -\sigma_i^2 + G^{i+1}(\pi) = -\sigma_i^2 + \frac{1}{2} \sigma_i \theta_i \). Thus \( f^i \) is strictly concave on \( \mathcal{R} \), and therefore admits a unique maximum \( \pi^*_i \), where \( f^{i+1}(\pi^*_i) = 0 \). Thus \( \pi^*_i \sigma_i - \theta_i = \frac{G^{i+1}(\pi)}{\sigma_i} \), which gives \( \pi^*_i = \frac{\theta_i}{\sigma_i} + \frac{G^{i+1}(\pi)}{\sigma_i^2} \) i.e. \( h(\pi^*_i) = 0 \). The result follows from the fact that, for each \( t \in [0, T] \), \( \max_{\pi \in \mathcal{A}_t} f(\pi) = f(\pi^*_i) \).

Note that \( h(\pi^*_i) = -1 - \frac{\sqrt{1 - q^2}}{q} \int_0^t \frac{\sigma_i^2 (\pi \sigma_i + \theta_i)}{\sigma_i^2 (\pi \sigma_i - \theta_i)} \). Hence \( h(\pi^*_i) \) is strictly decreasing function of \( \pi_i \). It follows that there exists a unique admissible optimal portfolio \( \pi^*_i \in [0, 1] \) such that \( h'(\pi^*_i) = 0 \) when \( h(0) > 0 \) and \( h(1) < 0 \).

(b) Since \( G^i(\pi) \) exists, then by part (a) we have an optimal portfolio \( \pi^{i+1} \) given by \( \pi^{i+1} = \frac{\theta_i}{\sigma_i} + \frac{G^{i+1}(\pi)}{\sigma_i^2} \), and \( \max_{\pi \in \mathcal{A}_t} f(\pi) = f(\pi^{i+1}) \). Assume that \( \pi^{i+1} \in \mathcal{A}_t \). Then \( E f^i(\pi) dt \leq E f^i(\pi^{i+1}) dt \), whence \( \max_{\pi \in \mathcal{A}_t} E f^i(\pi) dt = E f^i(\pi^{i+1}) dt \). Therefore by (13) with \( V_n = x \), we get

\[
\begin{align*}
& u(x) = \max_{\pi \in \mathcal{A}_t} E \log V_n \\
& = \log x + \frac{1}{2} \int_0^T (\theta_i^2) dt + \max_{\pi \in \mathcal{A}_t} E \int_0^T f^i(\pi') dt \\
& = \log x + \frac{1}{2} \int_0^T (\theta_i^2) dt + E \int_0^T f^i(\pi') dt.
\end{align*}
\]

Proof of Proposition 1

Recall that the optimal portfolio \( \pi^* \) for any investor is given by Theorem 1, as

\[
\pi^* = \frac{\theta}{\sigma} + \frac{G(\pi^*)}{\sigma^2} = \pi^* - \frac{G(\pi^*)}{\sigma^2} + \frac{G(\pi^*)}{\sigma^2} = \pi^* + \frac{G(\pi^*)}{\sigma^2 - G(\pi^*)}
\]

By the Mean Value Theorem there exists \( \psi_\pi \) between \( \pi^* \) and \( \pi \) such that

\[
G(\pi^*) = G(\pi) + (\pi^* - \pi)G'(\psi_\pi).
\]

Thus \( \pi^* - \pi = \frac{G(\pi^*)}{\sigma^2} = \frac{1}{\sigma^2} (G(\pi) + (\pi^* - \pi)G'(\psi_\pi)) \). Rearranging yields

\[
\pi^* - \pi = \frac{G(\pi)}{\sigma^2 - G(\psi_\pi)} = \pi^* + \frac{G(\pi)}{\sigma^2 - G(\psi_\pi)}
\]

Proof of Theorem 2

Let \( x > 0, i \in \{0, 1\} \) and \( T \to \infty \). From Theorem 3.1 of Guasoni (2006), we have

\[
u_{t,i} \sim \log x + \frac{1}{2} \int_0^T \left( \frac{\mu_i}{\sigma_i} \right) dt + \frac{\nu_T}{4} (1 - p)(1 - (-1)^{t+1})p.
\]

Since \( f(\pi) \) is continuous, then by L'Hospital's rule

\[
\lim_{T \to \infty} \frac{\nu_{t,i}(x)}{T} = \lim_{T \to \infty} \frac{1}{T} \int_0^T f^i(\pi') dt = \lim_{T \to \infty} \mathbb{E} \int_0^T f^i(\pi') dt = \varphi_{\pi^i}(i).
\]

Thus \( u_{t,i}(x) \sim T \varphi_{\pi^i}(i) \).

Proof of Theorem 3

(a) We now assume that \( k = 3 \), whence \( \int_0^T e^{\nu_T} - 1^3 \nu_T dt < \infty \), and so \( G^3(\pi) \) exists on \([0, 1]\). Expanding \( G(\pi) \) at \( \pi = 0, \) yields
\[ G(x) = G(0) + \alpha G'(0) + \frac{1}{2} \sigma^2 G''(0) + \frac{1}{2} \sigma^2 G''\left(\psi_1\right) \psi_1 \in (0, x). \]

With error term \( R_1(x) \approx \frac{1}{2} \sigma^2 G''\left(\psi_1\right), \) we have quadratic approximation of \( G(x) \)
\[ G(x) = \alpha G(0) + \frac{1}{2} \sigma^2 G'\left(\psi_1\right) + R_1(x), \]

Thus \( G(x) \approx M_1 x - \frac{1}{2} M_2 x^2, \) whence \( G(x) \approx \eta_0 x - M_1 \eta_1 + \frac{1}{2} M_2 \eta_2 \).

So the optimal portfolio becomes (cf Proposition 1)
\[ \pi = x + \frac{G'(x)}{\sigma^2} \approx x + \frac{M_1 x - \frac{1}{2} M_2 x^2}{\sigma^2 + M_2}. \]

Thus, the excess asset holdings due to jumps is \( \pi_2 \approx \pi - x \approx x - \frac{M_1 x + \frac{1}{2} M_2 x^2}{\sigma^2 + M_2}. \)

(b) Expanding \( G(\pi) \) at \( x \), there exists \( \eta_2 \) such that \( \pi = x + \frac{G'(x)}{\sigma^2} + \frac{1}{2} \pi_2 \) such that
\[ G(\pi) = G(x) + (\pi - x) G'(x) + \frac{1}{2} (\pi - x)^2 G''(\eta_2) \]
\[ \approx G(x) + (\pi - x) G'(x) - \frac{1}{2} M_2 (\pi - x)^2. \]

We now compute the integrand \( f(\pi) \) in the excess utility formula \( E \int_0^T f(\pi) d\tau \), which is defined by Eq. (14). Under quadratic approximation
\[ f(\pi) = G(x) - \frac{1}{2} \sigma^2 (\pi - x)^2 \approx G(x) + \frac{1}{2} \sigma^2 (\pi - x)^2 \]
\[ \approx G(x) + \frac{1}{2} \sigma^2 (\pi - x)^2 - \frac{1}{2} \pi_2. \]

whence
\[ f(\pi) = M_1 x - \frac{1}{2} M_2 x^2 + \frac{1}{2} (M_1 - M_2 x)^2 = \frac{1}{2} M_2 x^2 + \frac{1}{2} M_2 \pi_2 + \frac{1}{2} M_2 \pi_2 \]
\[ = A \pi_2 + B \pi + C = \theta (\pi - M_1, M_2). \]

Therefore, by the definition of \( u_{i,d}(x) \), we get (16).

(c) By part (b), the excess optimal utility due to the jumps for the ith investor is given by
\[ u_{i,d}(x) = E \int_0^T f(\pi) d\tau \approx E \int_0^T f(\pi) d\tau \approx \int_0^T f(\pi) d\tau \approx \int_0^T \left( A \pi_2 + B \pi + C \right) d\tau. \]

(18)

Since \( \pi = \frac{\mu}{\sigma} = \frac{\mu_1}{\sigma_1} + \mu_1 \), and with \( E(\pi_1) = 0 \), then \( E(\pi_2) = \frac{\mu_1}{\sigma_1} \).

Therefore
\[ E(\pi_2) = \frac{\mu_1}{\sigma_1} + 2 \frac{\mu_1}{\sigma_1} E(\pi_2) + E(\pi_2) = \frac{\mu_1}{\sigma_1} + E(\pi_2). \]

So by (18), we have
\[ u_{i,d}(x) \approx \int_0^T \left( A \pi_2 + B \pi_1 + C_1 \right) d\tau \approx \int_0^T \left( A \pi_2 + B \pi_1 + C_1 \right) d\tau. \]

(19)

Note that as \( t \to \infty \), \( E(\pi_2)^2 \to \frac{1}{2} (1 - p)(1 + (-1)^{i+1}p). \) By (19), as \( T \to \infty \), it follows that
\[ u_{i,d}(x) \approx \int_0^T Q \left( \frac{\mu_1}{\sigma_1}, \sigma_1, M_1, M_2 \right) dt + \int_0^T A \pi_2 d\tau. \]

(20)

From Theorem 2, the total optimal asymptotic utility of the ith investor is:
\[ u_{i,d}(x) = u_{i,d}^c(x) + u_{i,d}^l(x). \]

By Theorem 3.1 in Guasoni (2006), we know that
\[ u_{i,d}^l(x) \sim \log x + \frac{1}{2} \int_0^T \left( \frac{\mu_1}{\sigma_1} \right)^2 dt + \frac{T}{4} (1 - p)(1 + (-1)^{i+1}p). \]

(21)

By adding (20) and (21), we get (17). (d) The excess asymptotic optimal logarithmic utility of the informed investor is
\[ u_{i,d}^l(x) \sim \log x + \frac{1}{2} \int_0^T \left( \frac{\mu_1}{\sigma_1} \right)^2 dt + \frac{T}{4} (1 - p)(1 + (-1)^{i+1}p). \]

An explicit formula for \( \phi_{-i}(i) \)

We can give an explicit formula for \( \phi_{-i}(i) \) of Theorem 2 under quadratic approximation.

Corollary 1. For \( i \in \{0, 1\} \), under quadratic approximation of \( G \), we have
\[ \phi_{-i}(i) \sim \int_0^T Q \left( \frac{\mu_1}{\sigma_1} \right) dt + \frac{1}{2} A_{\infty} (1 - p)(1 + (-1)^{i+1}p). \]

Proof. From Theorem 2, as \( T \to \infty \), we have
\[ u_{i,d}(x) \sim \log x + \frac{1}{2} \int_0^T \left( \frac{\mu_1}{\sigma_1} \right)^2 dt + \frac{T}{4} (1 - p)(1 + (-1)^{i+1}p) + T \phi_{-i}(i). \]

By Theorem 3, under quadratic approximation,
\[ u_{i,d}(x) \sim \log x + \frac{1}{2} \int_0^T \left( \frac{\mu_1}{\sigma_1} \right)^2 dt + \frac{T}{4} (1 - p)(1 + (-1)^{i+1}p) + T \phi_{-i}(i). \]

Thus, under quadratic approximation, we find
\[ T \phi_{-i}(i) \sim \int_0^T Q \left( \frac{\mu_1}{\sigma_1} \right) dt + \frac{T}{4} (1 - p)(1 + (-1)^{i+1}p) \]
\[ - \frac{1}{2} A_{\infty} (1 - p)(1 + (-1)^{i+1}p) T, \]

from which the result follows.

References
