(1) (15 pts.) Complete the following definitions.
(a) We say that \( x \) is congruent to \( y \) modulo \( m \) if . . .
(b) A field is . . .
(c) An element \( x \) in a ring \( R \) is called a unit if . . .

(2) (15 pts.) Give examples of the following, in \( \mathbb{Z}/36 \).
(a) List three units and for each, find the order.
(b) List three zero divisors and for each, give a complementary zero divisor.
(c) Find one non-zero nilpotent element and find its index of nilpotency.

(3) (5 pts. each) Give an example of each of the following. Proofs are not required, nevertheless, be specific and describe your examples carefully. If an example does not exist, say so.
(a) A ring which is non-commutative.
(b) An infinite field.
(c) A finite field with 83 elements.

(4) (5 pts. each) Find all solutions for \( x \) modulo 84.
(a) \( 156x \equiv 29 \pmod{84} \)
(b) \( 164x \equiv 24 \pmod{84} \)
(c) \( 155x \equiv 25 \pmod{84} \)

(5) (5 pts. each) True or false? If true, give a proof. If false, give a counterexample.
(a) If \( ra \equiv rb \pmod{m} \) and \( m \) does not divide \( r \), then \( a \equiv b \pmod{m} \).
(b) If \( d \mid m \) and \( a \equiv b \pmod{m} \), then \( a \equiv b \pmod{d} \).
(c) If \( d \mid m \) and \( a \equiv b \pmod{d} \), then \( a \equiv b \pmod{m} \).
(d) If \( x \) and \( y \) are units in a ring \( R \), then \( xy \) is also a unit in \( R \).

(6) (10 pts. each) Prove the following.
(a) If \( m > 1 \) and \( m \) is a composite number, then \( \mathbb{Z}/m \) is not a field.
(b) If \( p \) is a prime number, then \( \mathbb{Z}/p \) is a field.