(1) (15 pts.) Complete the following definitions.
   (a) We say that \(a\) divides \(b\) if . . .
   (b) We say that \(d\) is the greatest common divisor of \(a\) and \(b\) if . . .
   (c) We say that \(a\) is congruent to \(b\) modulo \(m\) if . . .

(2) (5 pts) State without proof: The Division Algorithm.

(3) (10 pts.)
   (a) Apply the Euclidean Algorithm to compute the greatest common divisor of 258 and 585.
   (b) Illustrate Bezout’s Identity by writing the answer in part (a) as a linear combination of 258 and 585.

(4) (10 pts.) Give examples of the following. No proofs required.
   (a) numbers \(a\) and \(b\) such that \([a, b] = 75\)
   (b) a set \(S\) and an equivalence relation \(\sim\) on \(S\).

(5) (15 pts.) True or false? If true, give a proof. If false, give a counterexample.
   (a) If \(a \mid c\) and \(b \mid c\), then \(ab \mid c\).
   (b) If \((a, c) = d\) and \((b, c) = d\), then \((ab, c) = d\).
   (c) If \((a, b) = 1\) and \((b, c) = 1\), then \((a, c) = 1\).

(6) (15 pts.) Let \(x\) be a variable. Use Mathematical Induction to prove that
   \[
   \frac{x^{n+1} - 1}{x - 1} = x^n + x^{n-1} + \cdots + x + 1
   \]
   for all \(n \geq 1\).

(7) (15 pts.) From the definitions, prove: If \(ax + by = 1\), then \((a, b) = 1\).

(8) (15 pts.) From Bezout’s Identity, prove: If \((a, b) = 1\) and \(a \mid c\), and \(b \mid c\), then \(ab \mid c\).