Do not simplify your answers. Some answers will involve fractions, $\pi$, etc.

1. (20 pts.) Let $D$ denote the solid region inside both the ellipsoid $4x^2 + 4y^2 + z^2 = 64$ and the cylinder $x^2 + y^2 = 4$. Express the integral

$$\iiint_D (x^2 + y^2) dV$$

as an iterated integral in

(a) rectangular coordinates.
(b) cylindrical coordinates.

Do not evaluate.

2. (15 pts.) Let $R$ be the triangular region in the $(x,y)$-plane with vertices $(0,0)$, $(-1,0)$, and $(0,2)$. Let $S$ be the part of the surface $z = 2 + 3x^2 + 2y^3$ that lies above $R$.

(a) Draw a sketch in the $(x,y)$ plane, clearly showing the region $R$.
(b) Let $E$ be the solid region below $S$ and above $R$. Write an iterated integral for the volume of $E$. Do not evaluate.
(c) Write an iterated integral whose value is equal to the surface area of $S$. Do not evaluate.

3. (25 pts.) Let $S$ denote the part of the paraboloid $z = a^2 - x^2 - y^2$ that lies above the circle $y^2 = x(a-x)$, where $a > 0$ is a constant. Let $D$ denote the solid region under $S$ and above the $(x,y)$-plane.

(a) Write iterated integrals in rectangular coordinates for:
   (i) the volume of $D$.
   (ii) the surface area of $S$.
   Do not evaluate.
(b) Write both of the integrals of part (a) as iterated integrals in cylindrical coordinates. Do not evaluate.

4. (25 pts.) Let $R$ be the region in the $(x,y)$-plane bounded by $y = x^2$ and $y = x + 2$. The density function is $\rho(x,y) = x$.

(a) Draw a sketch, clearly showing the region $R$.
(b) Find $M$, the mass of $R$.
(c) Find $\bar{x}$, the $x$ coordinate of the centroid of $R$.

5. (15 pts.) Let $E$ be the solid region inside between the two concentric spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 = 36$. Express the integral

$$\iiint_E x^2 + y^2 dV$$

as an iterated integral in spherical coordinates. Evaluate the integral.