Do not simplify your answers. Some answers will involve square roots, fractions, \(\pi\), \(e\), etc..

(1) (20 pts.) For the function
\[ f(x, y) = x \ln(xy + 7) + y \]
and the point \( P_0 = (3, -2) \), do the following.
(a) Find \( \nabla f \), the gradient function.
(b) Find \( \nabla f(P_0) \), the gradient of \( f \) at \( P_0 \).
(c) Find the linearization of \( f \) at the point \( P_0 \).
(d) Use differentials to approximate \( f(3.01, -2.01) \).

(2) (20 pts.) Let \( P_0 = (-1, 2) \), \( \mathbf{v} = (-4, 3) \), and let \( f \) be the function
\[ f(x, y) = \frac{x}{(x + y)^2} \]
(a) Find \( \nabla f \), the gradient function.
(b) Find \( \nabla f(P_0) \), the gradient of \( f \) at \( P_0 \).
(c) Find the maximum rate of change of \( f \) at \( P_0 \).
(d) Find the directional derivative of \( f \) at \( P_0 \) in the direction of \( \mathbf{v} \).

(3) (20 pts.) Answer the following, for the point \( P_0 = (\pi/3, 2) \) and the function
\[ f(x, y) = \tan \frac{x}{y} \]
(a) Find the gradient function \( \nabla f \).
(b) Find the gradient of \( f \) at \( P_0 \).
(c) Find the equation of the tangent plane at the point \( P_0 \).
(d) Find the equation of the normal line at the point \( P_0 \).

(4) (20 pts.) For the function
\[ f(x, y) = xy^2 - 2x^3 - y^2 + 2x^2 \]
find all critical points. Classify each critical point as a local maximum, local minimum or saddle point.

(5) (20 pts.) Use the method of Lagrange to find the point on the plane
\[ x - 2y + 3z = 8 \]
that is closest to the point \( P_0 = (-1, 1, 3) \).