Do not simplify your answers. Some answers will involve square roots, fractions, \(e\), \(\pi\), et cetera. Unless specified otherwise, always assume up means in the \(z\)-direction, and positive orientation is with respect to the right-hand rule.

(1) (20 pts.) Items (a) – (d) refer to the two vectors
\[
\mathbf{u} = (1, -3, -4) \\
\mathbf{v} = (-1, 1, 2)
\]

(a) Find \(\mathbf{u} \cdot \mathbf{v}\).
(b) Find \(\mathbf{u} \times \mathbf{v}\).
(c) Find \(\text{proj}_\mathbf{v} \mathbf{u}\), the vector projection of \(\mathbf{u}\) onto \(\mathbf{v}\).
(d) Find the area of the parallelogram determined by \(\mathbf{u}\) and \(\mathbf{v}\).

(2) (20 pts.) For the parametric space curve \(C\) defined by
\[
\mathbf{r}(t) = (e^t, t + 1, t^2), \quad 0 \leq t \leq 1,
\]
answer the following.
(a) Set up, but do not evaluate, an integral giving the length of \(C\).
(b) Evaluate the line integral
\[
\int_C y\,dx + x\,dy + dz
\]

(3) (30 pts.) Let \(P_0 = (1, 1), \mathbf{v} = (-1, 1)\), and let \(f\) be the function
\[
f(x, y) = 3x - e^{x^2} + y^2
\]
(a) Find \(\nabla f\), the gradient function of \(f\).
(b) Find \(\nabla f(P_0)\), the gradient of \(f\) at \(P_0\).
(c) Find the linearization of \(f\) at the point \(P_0\).
(d) Find the directional derivative of \(f\) at \(P_0\) in the direction of \(\mathbf{v}\).

(4) (20 pts.) For the function
\[
f(x, y) = (x^2 - x)y + y^2
\]
find all critical points. Classify each critical point as a local maximum, local minimum or saddle point.

(5) (10 pts.) Use the method of Lagrange to find the minimum and maximum values of
\[
f(x, y) = 2x - 3y - 1
\]
subject to the constraint
\[
x^2 + y^2 \leq 4.
\]
(6) (30 pts.) Let \( R \) be the square region in \( \mathbb{R}^2 \) defined by
\[-1 \leq x \leq 1 \text{ and } -1 \leq y \leq 1\]
Let \( S \) be the portion of the surface
\[ z = (x - 1)(y + 1) \]
where the \((x, y)\) points are constrained to \( R \).
(a) Write the surface area of \( S \) as a double integral. Do not evaluate.
(b) For the vector field
\[ \mathbf{F} = \langle x - 1, y + 1, z \rangle \]
evaluate the upward flux of \( \mathbf{F} \) through \( S \).
(c) Let \( C \) denote the boundary curve of \( S \), oriented in the positive direction. Calculate the work done by the force field
\[ \mathbf{F} = \langle z, x, y \rangle, \]
on an object making one loop around \( C \).

(7) (40 pts.) Let \( E \) denote the solid cone in \( \mathbb{R}^3 \) defined by the inequalities
\[ \sqrt{x^2 + y^2} \leq z \leq 1 \]
(a) Set up, but do not evaluate, triple integrals for the volume of \( E \),
(i) in rectangular coordinates,
(ii) in cylindrical coordinates, and
(iii) in spherical coordinates.
(b) For the vector field
\[ \mathbf{F} = \langle x + xz, y + xy, z + yz \rangle, \]
do the following.
(i) Calculate the upward flux of \( \mathbf{F} \) through the top surface of \( E \).
(ii) Calculate the outward flux of \( \mathbf{F} \) through the boundary surface of \( E \).

(8) (20 pts.) Let \( C \) denote the triangle in \( \mathbb{R}^2 \) with vertices \((0, 0), (3, 1), (0, 1)\), with positive orientation. Let
\[ \mathbf{F} = \langle xy - x, y + 3x \rangle. \]
(a) Compute the work done by \( \mathbf{F} \) on an object making one loop around \( C \).
(b) Compute the outward flux of \( \mathbf{F} \) through \( C \).

(9) (10 pts.) Show that the line integral
\[ \int_C (y + 2x)dx + (2 + x + \cos y)dy \]
is path independent. Evaluate the line integral, if \( C \) is any path from \( A = (1, 0) \) to \( B = (2, \pi/2) \).