Write your complete solutions on the answer sheets. Do not write on this test paper.

(1) (20 points) Sketch the region $R$ in the $xy$-plane defined by the inequalities

$$x \geq 0, \quad y \leq 3x, \quad y \geq x(x^2 - 1).$$

Using double iterated integrals, do the following.

(a) Find the area of $R$.

(b) Find the $x$ coordinate of the centroid of $R$.

(2) (20 points) Let $R$ denote the triangle in the $xy$-plane with vertices $(0,0)$, $(1,0)$ and $(1,2)$.

(a) Sketch the region $R$.

(b) Express the integral $\iint_R e^{x^2 + y^2} \, dA$ using rectangular coordinates. Do not evaluate.

(c) Express the integral of part (b) using polar coordinates. Do not evaluate.

(3) (10 points) Let $S$ denote the solid in $xyz$-space defined by the inequalities

$$y \geq 0, \quad 0 \leq z \leq 4 - y - x^2.$$ 

Set up, but do not evaluate, a triple iterated integral for the volume of $S$.

(4) (10 points) Let $S$ denote the solid region in $xyz$-space defined by the inequalities

$$1 \leq x^2 + y^2 \leq 4, \quad 0 \leq z \leq 5 - x^2 - y^2.$$ 

Set up, but do not evaluate, a triple iterated integral for the volume of $S$.

(5) (30 points) Let $D$ denote the solid in $xyz$-space defined by the inequalities

$$0 \leq z \leq x^2 + y^2, \quad x^2 + y^2 \leq 1.$$ 

(a) Express the integral $\iiint_D z \, dV$ using rectangular coordinates. Do not evaluate.

(b) Express the integral of part (a) using cylindrical coordinates. Do not evaluate.

(c) Express the integral of part (a) using spherical coordinates. Do not evaluate.

(6) (10 pts.) Set up and evaluate the line integral of $f(x,y,z) = (x + y)z$ along the line segment from $(1,-1,1)$ to $(-1,0,-1)$. 