Instructions: Write your complete solutions on your answer paper. Do not write on this paper. No graphing calculators are allowed to be used. You are not required to simplify your answers.

(1) (10 pts.) Find a formula for the $n$th partial sum of the series

$$\sum_{n=1}^{\infty} \left( \frac{n}{2^{n-1}} - \frac{n + 1}{2^n} \right)$$

Determine whether the series converges or diverges. If it converges, find the sum.

(2) (20 pts.) For the following, determine whether the series converges or diverges. For each convergent series, find the sum.

(a) \(\sum_{n=0}^{\infty} \left( \frac{13}{5^n} - \frac{12}{7^n} \right)\)

(b) \(\sum_{n=1}^{\infty} \frac{n^n}{10^n}\)

(3) (10 pts.) Evaluate the integral \(\int_{2}^{\infty} \frac{dx}{x(\ln x)^2}\). Use your answer to determine whether the series \(\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}\) converges or diverges.

(4) (20 pts.) For the following, determine whether the series converges absolutely, converges conditionally, or diverges.

(a) \(\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2 + 3}}\)

(b) \(\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{10^n}\)

(5) (20 pts.) For each power series, find the interval of convergence.

(a) \(\sum_{n=0}^{\infty} \frac{(n + 1)(x - 1)^n}{n + 2}\)

(b) \(\sum_{n=0}^{\infty} \frac{x^n}{(n + 1)2^n}\)

(6) (10 pts.) For \(f(x) = \sqrt{x + 1}\), find the first four derivatives \(f'(x), f''(x), f'''(x), f^{(4)}(x)\). Use these to find the Taylor polynomial of order four for \(f(x)\) centered at \(a = 0\).

(7) (10 pts.) Start with \(\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots\) on the interval \((-1, 1)\). Find a power series for the following functions.

(a) \(\ln(1 + x)\)

(b) \(\ln(1 + x^5)\)

(c) \(\int \ln(1 + x^5) \, dx\)

(8) (5 pts. extra credit) Approximate \(\int_{0}^{0.5} \ln(1 + x^5) \, dx\) to within \(10^{-6}\).