Instructions: Some answers will involve square roots, fractions, logarithms, etc. Do not simplify. Do not use a calculator to round off.

(1) (25 pts.) The illumination of an object by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the light source. Two lights are separated by a distance of 8. The first light is 4 times stronger than the second. A point $P$ is on the line segment between the two lights.

$I_1 = 4$ \quad $P$ \quad $I_2 = 1$

(a) Find a formula for $I(x)$, the illumination at point $P$ as a function of $x$.
(b) Find $\lim_{x \to 0} I(x)$ and $\lim_{x \to 8} I(x)$.
(c) $I(x)$ has a minimum at some $0 < x < 8$. Find the $x$ value where it occurs.
(2) (25 pts.) For the function \( f(x) = 3x^5 - 10x^3 \), find the following.

(a) The \( x \)-intercepts.
(b) The first and second derivatives.
(c) The critical points and all possible inflection points.
(d) The intervals of increase or decrease.
(e) The intervals of concavity.
(f) The local extrema. Classify each as a maximum or minimum.
(g) The inflection points.
(h) Sketch the graph of \( y = f(x) \).
(3) (10 pts.) Find the limit.
\[ \lim_{{x \to \infty}} \left( 1 - \frac{5}{x} \right)^x \]

(4) (15 pts.)
(a) Find the linearization \( L(x) \) of the function \( f(x) = \frac{1}{{x^4}} \) at \( a = 2 \).
(b) Use part (a) to approximate \( f(2.1) \).
(5) (25 pts.) Find the absolute maximum and minimum values on the interval \([0, 3]\) for the function \(f(x) = \sqrt{x}(2 - x)\).