1. Consider a curve \( y = \sqrt{16 - x^2} \) from \((-2, 2\sqrt{3})\) to \((2, 2\sqrt{3})\). Determine the arc length and the surface area of the solid obtained by rotating \( y \) about the \( x \)-axis.

**Answer:**

| Arc length = \( \frac{4}{3} \pi \) units, and Surface Area = 32\( \pi \) sq. units |

**Solution:**

Here, \( y = \sqrt{16 - x^2} \implies \frac{dy}{dx} = -\frac{x}{\sqrt{16 - x^2}}. \)

Arc Length \( (L) = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \)

\[ = \int_{-2}^{2} \frac{4}{\sqrt{16 - x^2}} \, dx \]

Put \( x = 4\sin\theta \)

Differentiating both sides, we get \( dx = 4\cos\theta \, d\theta \)

Converting the limits over \( \theta \),

\( x = -2 \implies \theta = -\pi/6 \) and \( x = 2 \implies \theta = \pi/6 \)

So, \( L = \int_{-\pi/6}^{\pi/6} \frac{4(4\cos\theta)}{\sqrt{16 - 16\sin^2\theta}} \, d\theta \)

\[ = \int_{-\pi/6}^{\pi/6} \frac{4(4\cos\theta)}{4\cos\theta} \, d\theta \]

\[ = 4 \left[ \frac{\pi}{6} - \left( -\frac{\pi}{6} \right) \right] = \frac{4}{3}\pi. \]

The surface area \( (S_A) \) is

\[ S_A = \int_{-2}^{2} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \]

\[ = 2\pi \int_{-2}^{2} \sqrt{16 - x^2} \left( \frac{4}{\sqrt{16 - x^2}} \right) \, dx \]

\[ = 8\pi \int_{-2}^{2} \, dx \]

\[ = 32\pi \]