The standard normal distribution has three properties:

1. It’s graph is bell-shaped.
2. It’s mean is equal to 0 \((\mu = 0)\).
3. It’s standard deviation is equal to 1 \((\sigma = 1)\).

**Uniform Distribution**

A continuous random variable has a **uniform distribution** if its values are spread **evenly** over the range of probabilities.
Density Curve

A density curve is the graph of a continuous probability distribution. It must satisfy the following properties:

1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater. (That is, the curve cannot fall below the x-axis.)
Empirical Rule

About 68% of the area lies within 1 standard deviation of the mean.

About 95% of the area lies within 2 standard deviations.

About 99.7% of the area lies within 3 standard deviations of the mean.
Normal Distribution

μ-2σ  μ-σ  μ  μ+σ  μ+2σ
2.5%  13.5%  34%  34%  13.5%  2.5%
Determining Intervals

An instruction manual claims that the assembly time for a product is normally distributed with a mean of 4.2 hours and standard deviation 0.3 hour. Determine the interval in which 95% of the assembly times fall.

95% of the data will fall within 2 standard deviations of the mean.

\[ 4.2 - 2(0.3) = 3.6 \text{ and } 4.2 + 2(0.3) = 4.8. \]

95% of the assembly times will be between 3.6 and 4.8 hrs.
Standard Normal Distribution

The **standard normal distribution** is a normal probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.

Area $= 1$

$z$ Score
The cumulative area is close to 1 for $z$-scores close to 3.49.

The total area under the curve is one.

The cumulative area for $z = 0$ is 0.5000.

The cumulative area is close to 0 for $z$-scores close to –3.49.
Notation

\[
P(a < z < b) \quad \text{denotes the probability that the } z \text{ score is between } a \text{ and } b.\]

\[
P(z > a) \quad \text{denotes the probability that the } z \text{ score is greater than } a.\]

\[
P(z < a) \quad \text{denotes the probability that the } z \text{ score is less than } a.\]
### Finding Probabilities When Given $z$-scores using Table A-2

<table>
<thead>
<tr>
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<th>.02</th>
<th>.03</th>
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</table>
Guidelines to find area under the standard normal curve

1. Sketch the standard normal curve and shade the appropriate area under the curve.

2. Find the area by following the directions for each case shown.
   
a. To find the area to the left of $z$, find the area that corresponds to $z$ in the Standard Normal Table.

   2. The area to the left of $z$ is 0.8907.

1. Use the table to find the area for the $z$-score.
b. To find the area to the right of $z$, use the Standard Normal Table to find the area that corresponds to $z$. Then subtract the area from 1.

2. The area to the left of $z = 1.23$ is 0.8907.

3. Subtract to find the area to the right of $z = 1.23$:
   \[ 1 - 0.8907 = 0.1093. \]
c. To find the area between two z-scores, find the area corresponding to each z-score in the Standard Normal Table. Then subtract the smaller area from the larger area.

1. Use the table to find the area for the z-scores.

2. The area to the left of $z = 1.23$ is 0.8907.

3. The area to the left of $z = -0.75$ is 0.2266.

4. Subtract to find the area of the region between the two z-scores:
   $0.8907 - 0.2266 = 0.6641$. 
What is the area to the left of $Z=1.51$ in a standard normal curve?

Area is 93.45%
Example

\[ P(z < 1.27) = 0.8980 \]
### Table A-2 (continued)
Cumulative Area from the LEFT

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<th>$z$</th>
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<th>.02</th>
<th>.03</th>
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<td>.9236</td>
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<td>.9279</td>
<td>.9292</td>
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</tbody>
</table>
Example - Thermometers Reading

If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above \(-1.23\) degrees.

\[ P(z > -1.23) = 0.8907 \]

Probability of randomly selecting a thermometer with a reading above \(-1.23^\circ\) is 0.8907.

That is, 89.07% of the thermometers have readings above \(-1.23\) degrees.
Example - Thermometers Again

A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between \(-2.00\) and \(1.50\) degrees.

\[ P(z < -2.00) = 0.0228 \]
\[ P(z < 1.50) = 0.9332 \]
\[ P(-2.00 < z < 1.50) = 0.9332 - 0.0228 = 0.9104 \]

The probability that the chosen thermometer has a reading between \(-2.00\) and \(1.50\) degrees is 0.9104.

That is, If many thermometers are selected and tested at the freezing point of water, then 91.04\% of them will read between \(-2.00\) and \(1.50\) degrees.
Say we have GRE scores are normally distributed with mean 500 and standard deviation 100. Find the probability that a randomly selected GRE score is greater than 620.

- We want to know what’s the probability of getting a score 620 or beyond.

\[
\frac{620 - 500}{100} = 1.2 = z
\]

- \( p(z > 1.2) \)

- Result: The probability of randomly getting a score of 620 is \(~.12\)
Finding z Scores When Given Probabilities

Finding the 95th Percentile

(z score will be positive)
Finding the Lower 2.5% and Upper 2.5%

(One $z$ score will be negative and the other positive)
Find the $z$-score corresponding to a cumulative area of 0.9803.

$z = 2.06$ corresponds roughly to the 98th percentile.

Locate 0.9803 in the area portion of the table. Read the values at the beginning of the corresponding row and at the top of the column. The $z$-score is 2.06.
Find the z-score such that 45% of the area under the curve falls between \(-z\) and \(z\).

The area remaining in the tails is .55. Half this area is in each tail, so since \(\frac{.55}{2} = .275\) is the cumulative area for the negative \(z\) value and \(.275 + .45 = .725\) is the cumulative area for the positive \(z\). The closest table area is .2743 and the \(z\)-score is 0.60. The positive \(z\) score is 0.60.
Converting to a Standard Normal Distribution

\[ z = \frac{x - \mu}{\sigma} \]

(a) Nonstandard Normal Distribution

(b) Standard Normal Distribution
If a random variable, $x$ is normally distributed, the probability that $x$ will fall within an interval is equal to the area under the curve in the interval. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the probability that a person selected at random will have an IQ score less than 115.

To find the area in this interval, first find the standard score equivalent to $x = 115$.

$$z = \frac{150 - 100}{15} = 1$$
Probabilities and Normal Distributions

Find $P(z < 1)$. 

$P(z < 1) = 0.8413$, so $P(x < 115) = 0.8413$
Assume that the weights of the men are normally distributed with a mean of 172 pounds and standard deviation of 29 pounds. If one man is randomly selected, what is the probability he weighs less than 174 pounds?

\[ z = \frac{174 - 172}{29} = 0.07 \]

\[ P(x < 174 \text{lb.}) = P(z < 0.07) = 0.5279 \]
Q. Suppose the reaction times of teenage drivers are normally distributed with a mean of 0.53 seconds and a standard deviation of 0.11 seconds.

What is the probability that a teenage driver chosen at random will have a reaction time less than 0.65 seconds?

The goal is to find $P(x < 0.65)$.

1. The first step is to convert 0.65 to a standard score.

$z = (x - \text{mean}) / \text{standard deviation} = (0.65 - 0.53) / 0.11 = 1.09$

2. The problem now is to find $P(z < 1.09)$. This is a left tail problem. $P(z < 1.09) = 0.8621$ (see table)

Therefore, $P(x < 0.65) = 0.8621$
Find the probability that a teenage driver chosen at random will have a reaction time between 0.4 and 0.6 seconds.

The goal is to find \( P(0.4 < x < 0.6) \).

1. The first step is to convert 0.4 and 0.6 to the corresponding standard scores.

\[
z_1 = \frac{x \text{ - mean}}{\text{standard deviation}} = \frac{0.4 - 0.53}{0.11} = -1.18
\]

\[
z_2 = \frac{x \text{ - mean}}{\text{standard deviation}} = \frac{0.6 - 0.53}{0.11} = 0.64
\]

2. The problem now is to find \( P(-1.18 < z < 0.64) \). This is a "between" problem.

\[
P(-1.18 < z < 0.64) = P(z < 0.64) - P(z < -1.18)
\]

= 0.7389 - 0.1190 (see table)

= 0.6199

Therefore, \( P(0.4 < x < 0.6) = 0.6199 \)
What is the probability that a teenage driver chosen at random will have a reaction time greater than 0.8 seconds?

- The problem is to find $P(x > 0.8)$.

1. The first step is to find the corresponding standard score.

$$z = \frac{x - \text{mean}}{\text{standard deviation}} = \frac{0.8 - 0.53}{0.11} = 2.45$$

2. The problem now is to find $P(z > 2.45)$. This is a right tail problem.

$$P(z > 2.45) = 1 - P(z < 2.45) = 1 - 0.9929 \text{ (see table below)} = 0.0071$$

- Therefore, $P(x > 0.8) = 0.0071$
Example – Lightest and Heaviest

Use the data from the previous example to determine what weight separates the lightest 99.5% from the heaviest 0.5%?

\[ x = \mu + (z \cdot \sigma) \]
\[ x = 172 + (2.575 \cdot 29) \]
\[ x = 246.675 (247 \text{ rounded}) \]

The weight of 247 pounds separates the lightest 99.5% from the heaviest 0.5%.
Central Limit Theorem
Sampling Distributions

A sampling distribution is the probability distribution of a sample statistic that is formed when samples of size $n$ are repeatedly taken from a population. If the sample statistic is the sample mean, then the distribution is the sampling distribution of sample means.

The sampling distribution consists of the values of the sample means, $\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4, \bar{X}_5, \bar{X}_6, \ldots$
Sampling Distribution

• Summarize the information in the previous table to obtain the sampling distribution of the sample mean and the sample minimum:

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<tr>
<th>$\bar{x}$</th>
<th>$P(\bar{x})$</th>
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<tr>
<td>1.5</td>
<td>2/16</td>
</tr>
<tr>
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<td>3/16</td>
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</tr>
<tr>
<td>4.0</td>
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</table>

Histogram: Sampling Distribution of the Sample Mean
Sampling Distribution of the Sample Minimum:

<table>
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<tr>
<th>$m$</th>
<th>$P(m)$</th>
</tr>
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<td>3</td>
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<tr>
<td>4</td>
<td>1/16</td>
</tr>
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</table>

Histogram: Sampling Distribution of the Sample Minimum:
Consider the set \{1, 2, 3, 4\}.

1) Make a list of all samples of size 2 that can be drawn from this set (Sample with replacement).

2) Construct the sampling distribution for the sample mean for samples of size 2.

3) Construct the sampling distribution for the minimum for samples of size 2.

4) Construct the sampling distribution for the range for samples of size 2.
This table lists all possible samples of size 2, the mean, the minimum, the range and the probability of each sample occurring (all equally likely) respectively.

<table>
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<tr>
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<th>minimum</th>
<th>range</th>
<th>Probability</th>
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<td>4</td>
<td>0</td>
<td>1/16</td>
</tr>
</tbody>
</table>
A genetic experiment involves a population of fruit flies consisting of 1 male named Mike and 3 females named Anna, Barbara, and Chris. Assume that two fruit flies are randomly selected with replacement.

1. Use a table to describe the sampling distribution of the proportion of females.

2. Find the mean of the sampling distribution.

Assume the name of the fruit flies by the initials. Here the sample space is \{mm, ma, mb, mc, am, aa, ab, ac, bm, ba, bb, bc, cm, ca, cb, cc\}.

<table>
<thead>
<tr>
<th>Proportion of female (x)</th>
<th>Probability(P(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (all male)</td>
<td>1/16</td>
</tr>
<tr>
<td>0.5 (1 male 1 female)</td>
<td>6/16</td>
</tr>
<tr>
<td>1 (all female)</td>
<td>9/16</td>
</tr>
</tbody>
</table>
1. The random variable $x$ has a distribution (which may or may not be normal) with mean $\mu$ and standard deviation $\sigma$.

2. Simple random samples all of size $n$ are selected from the population. (The samples are selected so that all possible samples of the same size $n$ have the same chance of being selected.)
Let’s start small and draw 6 samples randomly from the uniform population (population that has uniform distribution).

Each sample has only 5 subjects (N=5). The central limit theorem requires that all samples must have the same sample size. The distribution of scores in each of these samples is presented below.

The sample mean is shown in blue.
Distribution of sample means
Conclusions:

1. The distribution of sample $\bar{x}$ will, as the sample size increases, approach a normal distribution.

2. The mean of the sample means is the population mean $\mu$.

3. The standard deviation of all sample means is $\sigma/\sqrt{n}$.
1. For samples of size $n$ larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets closer to a normal distribution as the sample size $n$ becomes larger.

2. If the original population is *normally distributed*, then for *any* sample size $n$, the sample means will be normally distributed (not just the values of $n$ larger than 30).
Notation

the mean of the sample means

$$\mu_{\bar{x}} = \mu$$

the standard deviation of sample mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(often called the standard error of the mean)
Example – Water Taxi Safety

Use the Chapter Problem. Assume the population of weights of men is normally distributed with a mean of 172 lb and a standard deviation of 29 lb.

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

b) Find the probability that 20 *randomly selected* men will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).
Example – cont

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

\[ z = \frac{175 - 172}{29} = 0.10 \]

\[ \mu = 172 \quad (\sigma = 29) \]

\[ x = 175 \]
b) Find the probability that 20 randomly selected men will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

\[ z = \frac{175 - 172}{\frac{29}{\sqrt{20}}} = 0.46 \]

\[ \mu_x = 172 \]

\[ (\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{29}{\sqrt{20}} \approx 6.4845971) \]

(b)
Example - cont

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

\[ P(x > 175) = 0.4602 \]

b) Find the probability that 20 *randomly selected men* will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

\[ P(\bar{x} > 175) = 0.3228 \]

It is much easier for an individual to deviate from the mean than it is for a group of 20 to deviate from the mean.
Correction for a Finite Population

When sampling without replacement and the sample size $n$ is greater than 5% of the finite population of size $N$ (that is, $n > 0.05N$), adjust the standard deviation of sample means by multiplying it by the finite population correction factor:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

finite population correction factor

Note that when $n = N$, the fpc is zero and the variance of the sampling distribution becomes zero. Also if $N$ is much larger than $n$, fpc is approximately one.

For example; if $N = 10,000$ and $n = 50$, then $fpc = \frac{9950}{9999} = 0.995 = 1$ (approx.).

Of course, if fpc is one then $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, which is somewhat easier to remember. As a general rule, The fpc may be ignored whenever the sample is less than 5% of the population,
The Central Limit Theorem

If a sample $n$ is greater than 30 is taken from a population with any type distribution that has a mean $\mu$ and standard deviation $\sigma$, then the sample means will have a normal distribution and standard deviation

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$
The Central Limit Theorem

If a sample of any size is taken from a population with a normal distribution with mean $= \mu$ and standard deviation $= \sigma$

the distribution of means of sample size $n$, will be normal with a mean $\mu_{\bar{X}} = \mu$

standard deviation

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$
The mean height of American men (ages 20-29) is $\mu = 69.2$ and $\sigma = 2.9$ inches. Random samples of 60 such men are selected. Find the mean and standard deviation (standard error) of the sampling distribution.

$\mu = 69.2$

$\sigma = 2.9$

Distribution of means of sample size 60, $\mu_X = \mu = 69.2$ will be normal.

Standard deviation

$$\sigma_X = \frac{2.9}{\sqrt{60}} = 0.3744$$
The mean height of American men (ages 20-29) is $\mu = 69.2"$. If a random sample of 60 men in this age group is selected, what is the probability the mean height for the sample is greater than 70"? Assume the standard deviation is 2.9".

Since $n > 30$ the sampling distribution of $\bar{X}$ will be normal

$\mu_{\bar{X}} = 69.2$

$$\sigma_{\bar{X}} = \frac{2.9}{\sqrt{60}} = 0.3744$$

Find the $z$-score for a sample mean of 70:

$$z = \frac{X - \mu}{\sigma_{\bar{X}}} = \frac{70 - 69.2}{0.3744} = 2.14$$
2.14

There is a 0.0162 probability that a sample of 60 men will have a mean height greater than 70".
During a certain week the mean price of gasoline in California was $1.164 per gallon. What is the probability that the mean price for the sample of 38 gas stations in California is between $1.169 and $1.179? Assume the standard deviation = $0.049.

Since \( n > 30 \) the sampling distribution of \( \bar{x} \) will be normal

Mean \( \mu_{\bar{x}} = \mu_{\bar{x}} = 1.164 \)

Standard deviation

\[
\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.049}{\sqrt{38}} = 0.0079
\]

Calculate the standard \( z \)-score for sample values of $1.169 and $1.179.

\[
z = \frac{1.169 - 1.164}{0.0079} = 0.63 \quad z = \frac{1.179 - 1.164}{0.0079} = 1.90
\]
Application Central Limit Theorem

\[ P(0.63 < z < 1.90) = 0.9713 - 0.7357 = 0.2356 \]

The probability is 0.2356 that the mean for the sample is between $1.169 and $1.179.