Florida Atlantic University
Mathematics Competition
2012

Name: ______________________________
Grade: ______
School: ______________________________
Home Address: __________________________

____________________________________
Email: _____________________________

I certify that the work presented herewith
is my own without the help of other people.

Signature: ______________________________
Date: _____________________________

Return your solutions with this form as a pdf emailed by **February 24, 2012** to
xzhang@fau.edu

or by regular mail (postmarked by February 24, 2012) to
Professor Xiaodong Zhang
Department of Mathematics
Florida Atlantic University
777 Glades Road,
Boca Raton, Florida, 33431-0991

Please submit only one entry. If you submit more than once, only the first entry will be
considered. If you send by both email and regular mail, only the regular mail entry will be
considered. Your submission will be acknowledged in the week of February 15.
Problem 1.

(1a) Is there a prime number $p$ for which the decimal expansion of $p!$ contains a string of exactly 2012 zeros at the end?

(1b) Given three odd square integers $x^2, y^2, z^2$, prove that there is an odd integer square $w^2$ such that $x^2 + y^2 + z^2 + w^2$ is the square of an integer.
Name:______________

**Problem 2.**
For $a < b$, it is known that $ax^2 + bx + c \geq 0$ for all real numbers $x$. Find the least possible value of $\frac{a+b+c}{b-a}$. 
Problem 3.
Define a sequence of real numbers \((x_n)\) by
\[ nx_n = 2(2n - 1)x_{n-1}, \quad x_1 = 2. \]
Prove that every \(x_n\) is an integer.
Name:__________________

Problem 4.
A infinite sequence of integers $a_1, a_2, \ldots, a_n, \ldots$ satisfies the condition that $a_m + a_n$ is divisible by $mn$ for all positive integers $m$ and $n$. Prove that $a_n = 0$ for every positive integer $n$. 
Problem 5.
Given a triangle $ABC$, the bisector of angle $A$ intersects the
circumcircle at $M$. Let $Y$ and $Z$ be the feet of the perpendicu-
lars from the incenter $I$ to $BM$ and $CM$. Suppose $IY + IZ = rac{1}{2}AM$, find $\angle BAC$. 
Problem 6.
Given a fixed point $A$ and a variable point $P$ on a circle $C$ with center $O$, let the line $AP$ intersect the circle again at $X$. If $Y$ is the reflection of $X$ in $OA$, prove that the line $PY$ intersects $OA$ at a fixed point.