Problem 1 (Halve, or triple and add one)  5 Points

Given a positive integer \( n_0 \), consider the sequence \( (n_0, n_1, \ldots) \) where for \( i \geq 0 \) we set

- \( n_{i+1} = \frac{n_i}{2} \), if \( n_i \) is even;
- \( n_{i+1} = 3n_i + 1 \), if \( n_i \) is odd.

Further, for a positive integer \( n_0 \), let the function \( f(n_0) \) be defined as

- \( f(n_0) = r \), if \( n_r = 1 \) and \( n_i \neq 1 \) for all \( i < r \) in the above sequence;
- \( f(n_0) = \infty \), if \( n_i \neq 1 \) for all \( i \geq 0 \).

(a) Write a Python function which on input a positive integer \( n_0 \) and a non-negative integer \( r \) outputs the values \( n_0, \ldots, n_r \).
(b) Write a Python program to compute the maximal value achieved by \( f \) when applied to all positive integers less than 1000, and compute this maximum.
(c) What is the value of \( f \) when being evaluated at your Z-number?

Problem 2 (Day of the week)  5 Points

Write a Python program that asks the user to input a date in the form “MMDDYYYY”, and then outputs the day of the week of this date. For instance, on input “01192010” your program should output “Tuesday”. Your program must be capable of handling all dates between January 1, 1900 and December 31, 2100 (including these two dates).

Problem 3 (Binary search)  5 Points

Write a program that can guess any integer in the range \( \{1, \ldots, 10^6\} \) by asking the user no more than 42 questions of the form “Is the number greater than \( x \)” or “Is the number less than \( x \)?”.

Good luck—and do not hesitate to ask questions!!