Calculus III (MAC 2313-17772)
Quiz #1–Solutions

1. Consider the points \( P(2, 6, 3) \), \( Q(-1, -5, 4) \) and \( R = (3, 0, 8) \).

(a) (15 points) Which of these points is closest to the \( yz \)-plane?

Solution
The distance of the point \( P(x, y, z) \) from the \( yz \)-plane is \( |x| \), so of the three points the one closest to the \( yz \)-plane will be the one with the smallest \( |x| \) value. This is \( Q \), so the answer is \( Q \).

(b) (10 points) Which of these points lies in the \( xz \)-plane?

Solution
A point lies in the \( xz \) plane if and only if its \( y \) coordinate is 0; thus \( R \) is the (only) point of the three on the \( xz \) plane.

Justify your answers.

2. (25=20+5 points) Show that the equation

\[ x^2 + y^2 + z^2 - 6x + 2y - 15 = 0 \]

represents a sphere and find its center and radius. What is the intersection of this sphere with the plane \( y = 1 \)?

Solution
Completing the squares, the equation can be written in the form

\[(x - 3)^2 + (y + 1)^2 + z^2 = 25,\]

which is the equation of a sphere of center \((3, -1, 0)\) and radius 5.
Setting \( y = 1 \), the equation becomes

\[(x - 3)^2 + z^2 = 21,\]

thus the intersection of the sphere with the plane \( y = 1 \) is a circle of radius \( \sqrt{21} \) centered at the point \( x = 3, z = 0 \) on the plane \( y = 1 \).

3. Let \( \mathbf{a} = (6, 2, 3), \quad \mathbf{b} = (-1, 5, -2) \).

Determine the following objects:

(a) (5 points) \( |\mathbf{a}| \). Solution

\[ |\mathbf{a}| = \sqrt{6^2 + 2^2 + 3^2} = 7. \]
(b) (5 points) $3a + 4b$. Solution

$$3a + 4b = (14, 26, 1).$$

(c) (5 points) $a \cdot b$. Solution

$$a \cdot b = -2.$$  

(d) (5 points) A unit vector $u$ in the same direction as $a$. Solution

$$u = \frac{1}{7}(6, 2, 3).$$

(e) (5 points) A vector $c$ such that $a - 3b + 2c = 0$. Solution

We should have $2c = 3b - a$, thus

$$c = \frac{3}{2}b - \frac{1}{2}a = \langle -\frac{9}{2}, \frac{13}{2}, -\frac{3}{2} \rangle.$$  

4. (25 = 20 + 5 points) Find the angle between the vectors $4i + 3j$ and $-8i - 6j$. Sketch the vectors.

Solution

If the angle is $\theta$, then

$$\cos \theta = \frac{(4i + 3j) \cdot (-8i - 6j)}{|4i + 3j| | -8i - 6j|} = \frac{-32 - 18}{5 \cdot 10} = -1.$$  

Thus $\theta = \pi$.

Each tickmark on the axes represents one unit.