Calculus III (MAC 2313-17772)
Homework #1
Due date: Monday, September 20, 2004.
(Seems very long, but most exercises are easy and can be done in a few minutes, or seconds)

Instructions For full credit, homework must be handed in by the end of the class on the day that it is due. Otherwise, the following penalties may be assessed.

- Homework handed in later on that same day: 5% reduction in the grade.
- Homework handed in past the due date, but before the end of the next class: Grade reduced by 20%.
- Homework handed in after the class following the due date is over, but within the same week: 50% reduction in the grade.
- Homework handed in past the week of the due date: 100% reduction in the grade.

In writing out your homework, make sure that each problem is clearly identified and appears as a unit. You should not start (say) problem 3, then interrupt to do (say) problem 4, return to problem 3 later. Your work should be easy to read. You should explain what you are doing. If I find a problem hard to read, I may simply ignore it. Moreover, I may decide not to grade every problem, just a selection. In that case, if I have difficulties finding one of the selected problems, I may assume that you didn’t do it.

Answers to the homework may be posted on some occasions. Whether I post answers or not, you should always feel free to see me to discuss your homework, and find out how the exercise should have been done (in case I marked you down).

1. Calculate the given quantity if
   \[ a = i + j - 2k \quad b = 3i - 2j + k \quad c = j - 5k \]
   (a) \( a \cdot b \).
   (b) \( a \times b \).
   (c) \( a \cdot (b \times c) \).
   (d) \( a \times (b \times c) \).
   (e) \( (a \times b) \times c \).
   (f) The angle between \( a \) and \( b \) (in radians, correct to 2 places after the decimal point).

2. Let \( \mathbf{u}, \mathbf{v} \) be in \( V_3 \); that is, vectors of 3 components. Show: \( \mathbf{u} \) and \( \mathbf{v} \) are orthogonal if and only if \( |\mathbf{u} + \mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 \).
3. Given the points \(A(1, 0, 1), B(2, 3, 0), C(-1, 1, 4),\) and \(D(0, 3, 2),\) find the volume of the parallelepiped with adjacent edges \(AB, AC,\) and \(AD.\)

4. Find the vector equation, parametric equations and the symmetric equations for the following lines.

   (a) The line through the points \((-1, 2, 3)\) and \((4, -3, 2).\)

   (b) The line through \((1, 1, -3)\) and perpendicular to the plane \(x - 2y + 3z = 5.\)

5. For the following lines, determine if they are parallel, skew or intersecting. If intersecting, determine their intersection point.

   (a) The lines of equations
   \[ \mathbf{r} = (3, 1, 5) + t(2, -1, 2) \]
   and
   \[ \mathbf{r} = (-2, 2, 4 + t)(3, 0, -1) \]

   (b) The lines of equations
   \[ \mathbf{r} = (1, 2, 3) + t(2, 3, 4) \]
   and
   \[ \mathbf{r} = (-1, 3, -5 + t)(6, -1, 2) \]

6. Find the equation of the following planes.

   (a) The plane through \((2, 1, 0)\) and perpendicular to the vector \(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}.\)

   (b) The plane through the points \((3, 2, 2), (2, -1, -1),\) and \((1, 2, -1).\)

   (c) The plane through \((1, 2, -2)\) that contains the line \(x = 2t, y = 3 - t, z = 1 + 3t.\)

7. Coordinate changes

   (a) The rectangular (Cartesian) coordinates of a point are \((2, 2, -1).\) Find the cylindrical and spherical coordinates of the point.

   (b) The cylindrical coordinates of a point are \((2\sqrt{5}, \pi/6, 4).\) Find the rectangular and spherical coordinates of the point.

   (c) The spherical coordinates of a point are \((6, \pi/4, \pi/3).\) Find the rectangular and cylindrical coordinates of the point.

8. Find the domain of the vector function.

   \[ \mathbf{r}(t) = \frac{t - 2}{t + 2} \mathbf{i} + \sin(t) \mathbf{j} + \ln(9 - t^2) \mathbf{k}. \]
9. Find the limit
\[ \lim_{t \to \infty} \left\langle \arctan t, \left( 1 + \frac{2}{t} \right)^t, te^{-3t} \right\rangle. \]

10. Show that the curve with parametric equation \( x = t \cos t, y = t \sin t, z = t^2 \) lies on the paraboloid \( z = x^2 + y^2 \) and use this fact to help sketch the curve.

11. Two particles travel along space curves
\( r_1(t) = \langle t, t^2, t^3 \rangle \quad r_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle. \)

Do the particles collide? Do their paths intersect?

12. Consider the plane curve of vector equation
\( r(t) = e^t \mathbf{i} + e^{-t} \mathbf{j}. \)

(a) Sketch the curve. It might help to figure out what equation is satisfied by the points \((x, y)\) on the curve; i.e., by the points \((x, y)\) such that \(x = e^t, y = e^{-t}\) for some \(t\). One can do this by eliminating \(t\) from the equations, getting a relation between \(x\) and \(y\).

(b) Find \(r'(t)\).

(c) Sketch the position vector \(r(t)\) and the tangent vector \(r'(t)\) for \(t = 0\). (Add this to the sketch of the curve).

13. Find the derivative of the following vector functions.

(a) \( r(t) = (\cos 3t, t, \sin 3t) \).

(b) \( r(t) = 2i + j + e^{-2t}k \).

(c) \( r(t) = t \mathbf{a} \times (\mathbf{b} + t \mathbf{c}) \), where \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) are constant vectors.

14. Find the parametric and symmetric equations for the tangent line to the curve given by
\( x = t^2 - 1, \quad y = t^2 + 1, \quad z = t + 1 \)
at the point \((0, 2, 0)\).

15. If \( r(t) = \langle t \cos t, t \sin t, e^t \rangle \), find \( r'(t), \mathbf{T}(t), r''(t), \) and \( r'(t) \times r''(t) \).

16. Find \( r(t) \) if \( r'(t) = \sin t \mathbf{i} - \cos t \mathbf{j} + 2t \mathbf{k} \) and \( r(0) = \mathbf{i} + \mathbf{j} + 2 \mathbf{k} \).