- **Aliquot function**. The *aliquot function* s takes a number greater than 1 to the sum of its proper divisors. For example, $s(10) = 1 + 2 + 5 = 8$. The fixed points of s are the **perfect numbers**; the points of period two are **amicable numbers**. A number n is **abundant** if $s(n) > n$ and **deficient** if $s(n) < n$.

  If $n > 1$, and $s(m) = n$, then $m \leq (n-1)^2$ so the set $s^{-1}(n)$ is finite. The set $s^{-1}(1)$ is the set of primes. If $s^{-1}(n)$ is empty, then we say that n is **untouchable**.

- **Aliquot sequences**. The sequence $s(n),\ s(s(n)),\ s(s(s(n))),\ \ldots$ is called the *aliquot sequence* of n. If it contains a perfect number (and, so, essentially ends there), then n is said to be an **aspiring number**. The numbers 25 and 95 are aspiring numbers: the aliquot sequence of 95 is 95, 25, 6, and 6 thereafter. It is unknown whether every aliquot sequence is finite—in particular, it is unknown if the aliquot sequence of 276 is finite.

- **Archimedean solids**. There are thirteen of these. The surface of an Archimedean solid is composed of different kinds of regular polygon, but each vertex has the same arrangement of polygons around it.

- **Emirp**. The word is "prime" spelled backwards. A number is an emirp if it is prime and its reversal is a *different* prime. So 13 is an emirp because 31 is also a prime, but 11 is not an emirp, it is a palindromic prime. Of course this depends on the base. An unstated base is equal to ten.

- **Euler totient function** is often called the φ-function. It takes n to the number of elements in the set $\{1, 2, \ldots, n\}$ that have no common factor with n other than 1.

- **Figurate numbers**. The n^{th} **triangular number** is the number

  $$1 + 2 + 3 + 4 + \cdots + n = n(n+1)/2$$

  The n^{th} **square number** is the number n^2. The n^{th} **pentagonal number** is the sum of the n^{th} square and the $(n-1)^{\text{st}}$ triangular number.

- **Gematria** is a numerological technique whereby numbers are assigned to letter and words. A standard way in English is to assign each letter its position in the alphabet. In Hebrew where letters have long stood for numbers, the eleventh letter stands for 20, the twelfth for 30, and so on until 100. The remaining three letters are assigned the values 200, 300, and 400. A similar technique is used in Greek.

- **Goldbach's conjecture** says that every even integer greater than 2 is the sum of two primes. It is one of the oldest unsolved problems in mathematics, going back at least to 1742.

- **Happy numbers**. Consider the function f that takes a number to the sum of the squares of its digits. A *happy number* is a number for which iterating f ends you up at 1, the unique fixed point of f. So 7 is happy because iterating f results in the sequence 49, 97, 130, 10, 1.

- **Lagrange's four-square theorem**: each positive integer is the sum of four integer squares. The theorem was stated in the *Arithmetica of Diophantus* around the year 250, translated into Latin by Bachet in 1621, and proved by Lagrange in 1770.

- **Mersenne primes** are primes of the form $2^n - 1$ for some n. The first few are $3 = 2^2 - 1$, $7 = 2^3 - 1$, $31 = 2^5 - 1$. In general, n must be prime, but that's not enough to make $2^n - 1$ prime because $2^{11} - 1 = 23 \times 89$.

- **Odd and even**. A number is **even** if it is divisible by 2. A number is **odd** if it is not even. It follows that a number is odd exactly when it is an even number plus one.

- **Palandromic numbers**. A palindromic number is a number that is equal to its reversal, like 121. A palindrome is a word, like *level*, or a phrase like *Madam, I'm Adam*, that reads the same forwards an backwards.

- **Platonic solids**. There are five of these. The surface of a Platonic solid is composed of a single kind of regular polygon, and the vertices all look the same.

  The surface of a **tetrahedron** consists of four equilateral triangles, the surface of a **cube** consists of six squares, the surface of an **octahedron** consists of eight equilateral triangles, the surface of a **dodecahedron** consists of **twelve** regular pentagons, and the surface of an **icosahedron** consists of twenty equilateral triangles.

- **Primes and composites**. A number p is **prime** if it is greater than 1 and its only divisors are 1 and p. A number is **composite** if it can be written as a product of two smaller numbers.

- **Pythagorean triples**. This is a triple of positive integers, $[a, b, c]$, such that $a^2 + b^2 = c^2$. It is called *primitive* if a, b, and c, have no common factor other than 1.