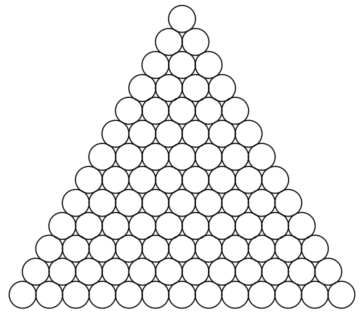
91 Ninety-One XCI



Corresponding ordinal: ninety-first.

The number 91 is the forty-sixth odd number and the sixty-sixth composite number.

As a product of primes: $91 = 7 \cdot 13$.

The number 91 has four divisors: 1, 7, 13, 91.

The number 91 is the sixty-ninth deficient number: $s(91) = 1 + 7 + 13 = 21 < 91$.

As a sum of four or fewer squares: $91 = 1^2 + 3^2 + 9^2 = 1^2 + 1^2 + 5^2 + 8^2 = 3^2 + 3^2 + 3^2 + 8^2 = 1^2 + 4^2 + 5^2 + 7^2 = 4^2 + 5^2 + 5^2 + 5^2$.

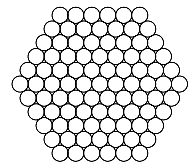
As a sum of nine or fewer cubes: $91 = 3 \cdot 1^3 + 3 \cdot 2^3 + 4^3 = 2 \cdot 1^3 + 2^3 + 3 \cdot 3^3 = 8 \cdot 2^3 + 3^3 = 3^3 + 4^3$.

As a difference of two squares: $91 = 46^2 - 45^2$.

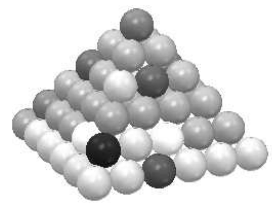
The number 91 appears in five Pythagorean triples: $[35, 84, 91]$, $[60, 91, 109]$, $[91, 312, 325]$, $[91, 588, 595]$, $[91, 4140, 4141]$. The second and the last are primitive.

As a sum of three odd primes: $91 = 3 + 5 + 83 = 3 + 17 + 71 = 3 + 29 + 59 = 3 + 41 + 47 = 5 + 7 + 79 = 5 + 13 + 73 = 5 + 19 + 67 = 5 + 43 + 43 = 7 + 11 + 73 = 7 + 13 + 71 = 7 + 17 + 67 = 7 + 23 + 61 = 7 + 31 + 53 = 7 + 37 + 47 = 7 + 41 + 43 = 11 + 13 + 67 = 11 + 19 + 61 = 11 + 37 + 43 = 13 + 17 + 61 = 13 + 19 + 59 = 13 + 31 + 47 = 13 + 37 + 41 = 17 + 31 + 43 = 17 + 37 + 37 = 19 + 19 + 53 = 19 + 29 + 43 = 19 + 31 + 41 = 23 + 31 + 37 = 29 + 31 + 31$.

The number 91 is the thirteenth triangular number because it is equal to $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13$. You can see this in the figure above. It is also a centered hexagonal number because it is equal to $1 + 6 + 12 + 18 + 24 + 30$.



Moreover, it is a pyramidal number because it is equal to $1 + 4 + 9 + 16 + 25 + 36$, the sum of the first six squares.



The number 91 is the smallest number that is both the sum of two cubes and the difference of two cubes: $3^3 + 4^3$ and $6^3 - 5^3$. The next two such numbers are 152 and 189 which also come from rewriting the equation $3^3 + 4^3 + 5^3 = 6^3$, this time as $152 = 3^3 + 5^3 = 6^3 - 4^3$ and $189 = 4^3 + 5^3 = 6^3 - 3^3$. The next cube after 6^3 that is the sum of three nonzero cubes is $9^3 = 1^3 + 6^3 + 8^3$.

The number 91 is the only composite two-digit numbers that passes the three standard quick primality tests for two-digit numbers greater than 11: it does not end in an even digit or in 5, the sum of its digits is not divisible by 3, and its digits are different.

The number 91 is the smallest *Fermat pseudoprime* to the base 3. That is, $3^{90} \equiv 1 \pmod{91}$, yet 91 is composite. The next two such numbers are 121 and 286. If p is a prime, then Fermat's little theorem says that $a^{p-1} \equiv 1 \pmod{p}$ for any number a not divisible by p. So a simple test for primality is to calculate a^{n-1} modulo n for some a not dividing n. If a number n fails such a test, then it is not prime.

As $91 = 7 \cdot 13$, it inherits mystical properties from both 7 and 13. If we multiply it by 11, which is the unique prime between 7 and 13, we get 1001. The primes 7, 11, and 13 form a *prime triple* because $13 - 7 = 6$ which is the closest the first and last of three consecutive odd primes can get.

The number 91 is the numerical value of the Hebrew word "amen".

If you have one U.S. coin of each denomination less than one dollar, you have 91 cents.