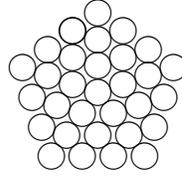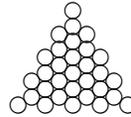# 31 Thirty-One XXXI

Corresponding ordinal: thirty-first.

The number 31 is a centered pentagonal number as you can see in the diagram above.

The number 31 is also the fifth centered triangular number:

The number 31 is the sixteenth odd number, the eleventh prime number, and the twenty-fourth deficient number.

The number $31 = 2^5 - 1$ is the third Mersenne prime. It is associated with the perfect number $2^4 \cdot 31 = 496$.

The reverse of 31 is 13 which is also a prime. So 31 is an emirp.

The number 31 is in the fifth twin-prime pair 29, 31.

As the sum of four squares: $31 = 1^2 + 1^2 + 2^2 + 5^2 = 2^2 + 3^2 + 3^2 + 3^2$.

As the sum of nine or fewer cubes: $31 = 4 \cdot 1^3 + 3^3$.

As a difference of two squares: $31 = 16^2 - 15^2$.

The number 31 appears in only one Pythagorean triple $[31, 480, 481]$, which is primitive, of course.

As a sum of three odd primes: $31 = 3 + 5 + 23 = 3 + 11 + 17 = 5 + 7 + 19 = 5 + 13 + 13 = 7 + 7 + 17 = 7 + 11 + 13$.

The number 31 is the smallest number that is the sum of the proper divisors of five different numbers: $s^{-1}(31) = \{32, 125, 161, 209, 221\}$.

There are exactly 31 numbers that cannot be written as the sum of distinct squares. They are 2, 3, 6, 7, 8, 11, 12, 15, 18, 19, 22, 23, 24, 27, 28, 31, 32, 33, 43, 44, 47, 48, 60, 67,

72, 76, 92, 96, 108, 112, 128.

The sum of the first 23 odd primes is 31^2. The sum of the first 31 odd primes is 43^2. (Prime Curios)

The number 31 is the smallest prime, and possibly the only one, that is the sum of the divisors of two different numbers, 16 and 25. In fact, $31 = 1+2+4+8+16 = 1+5+25$. (Number Gossip) No other numbers less than a million have this property

Note that $31 = 1 + 5 + 5^2 = 1 + 2 + 2^2 + 2^3 + 2^4$ is the sum of at least three successive powers, starting with 1, in two different ways. The only other number known to have this property is $8191 = 2^{13} - 1$, which is also a Mersenne prime. (Wells) It is equal to $1 + 2 + 4 + 8 + \cdots + 2^{12}$ and to $1 + 90 + 90^2$. These are the numbers that are *repunits*, with at least three digits, in two different bases. A quick search reveals that any other such number must have at least fifteen digits.

The number 31 is the sum of the first two primes raised to themselves: $2^2 + 3^3 = 31$. (Number Gossip)

The thirty-first President of the United States was Herbert Clark Hoover.

The thirty-first state to enter the Union was California.

The thirty-first largest state in the United States is Louisiana.

There are 31 days in January, March, May, July, August, October, and December.