

3 Three III



Corresponding ordinal: third.

The number 3 is called a *triangular number* because $3 = 1 + 2$, the sum of the first two positive numbers. Note the triangular pattern above. The number 1 is considered to be the first triangular number . . . is that because it is the sum of the first one positive numbers? What about 0, is that a triangular number? Is it the sum of the first zero positive numbers?

The number 3 is the second odd number, the second prime number, and the third deficient number.

The number $3 = 2^2 - 1$ is the first *Mersenne prime*, a prime of the form $2^n - 1$. The next two are $7 = 2^3 - 1$ and $31 = 2^5 - 1$, not $15 = 2^4 - 1$ because $15 = 3 \cdot 5$ is not prime.

Twin-prime pairs are pairs of primes that differ by 2. The number 3 appears in the first twin-prime pair 3, 5.

Every number can be written as the sum of four or fewer square numbers. For 3 we can write $3 = 1^2 + 1^2 + 1^2$.

Every number can be written as the sum of nine or fewer cubes. For 3 we can write $3 = 1^3 + 1^3 + 1^3$.

As the difference of two squares, $3 = 2^2 - 1^2$.

The *Pythagorean Theorem* says that, in a right triangle (a triangle with one angle equal to 90 degrees), the square of the longest side—called the *hypotenuse*—is equal to the sum of the squares of the other two sides. A triangle with sides of lengths 3, 4, and 5 has this property, because $3^2 + 4^2 = 5^2$. No right triangle, whose sides have integer lengths, can have a side of length 1 or 2. So this is the smallest such right triangle. The three numbers 3, 4, and 5, taken together, are called a *Pythagorean triple*. We will write this as $[3, 4, 5]$. Every number from 3 on appears in some Pythagorean triple.

Every number is the sum of 3 or fewer triangular numbers. When Gauss proved this, he wrote in his diary, “EYRHKA! num = $\Delta + \Delta + \Delta$ ”. That “Y” is an upper-case upsilon, and the “H” is an upper-case eta. Was he thinking of Archimedes?

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The most famous theorem saying that you can't do something says that you can't divide an arbitrary angle into 3 equal parts using just a straightedge and compass.

A closed polygon with 3 sides is called a *triangle*. That's what Euclid called it, a figure with three *angles*. Why do we all say that it is a figure with three *sides*?

The word is 3 dimensional, or so it appears.

The third President of the United States was Thomas Jefferson.

The first ten amendments of the U.S. Constitution are called the *Bill of Rights*. The third amendment says that "No soldier shall in time of peace be quartered in any house without the consent of the Owner, nor in time of war but in a manner to be prescribed by law."

The third state to enter the Union was New Jersey.

The third largest state in the United States is California.

The number 3 is often heard in the statement, "Two's company, three's a crowd."

In baseball, three strikes and you're out.

In many states of the United States, there is a "three strikes" law—if a person is convicted of three felonies the sentence must be very long, in some cases life in prison.

One time is once, two times is twice, and three times is thrice.

There used to be a British coin worth three pennies. It was called *threepence*.

The Threepenny Opera by Kurt Weill (1900-1950) features a character named "Mack the knife."

The Holy Trinity is the Father, the Son, and the Holy Ghost. Peter denied Christ 3 times.

Three Blind Mice is a famous nursery rhyme.