3.2 Basic Concepts

Slide 1

❖ **Random Experiment** an experiment whose outcome cannot be predicted with certainty, before the experiment is run.

❖**Event** Any collection of outcomes of a random experiment.

❖ **Simple Event** An outcome or an event that cannot be further broken down into simpler components.

❖ **Sample Space** Consists of all possible *simple* events. That is, the sample space consists of all outcomes that cannot be broken down any further.

Slide 2

Example:

Experiment	Sample of Event	Sample Space
Roll 1 die	{5} (simple event)	{1,2,3,4,5,6}
Roll 2 dice	{(3,4), (2,5), (1,6)} (not a simple event)	{(1,1), (1,2), …, (6,6)}

Notation for Probabilities

Slide 3

P - probability.

A, B*, and *C - events.

***P* (*A*)** - probability of event A occurring.

Basic Rules for Computing Probability

Slide 4

Rule 1: Relative Frequency Approximation of Probability

$$P(A) = \frac{\text{number of times A occurred}}{\text{number of times trial was repeated}}$$

Slide 5

Rule 2: Classical Approach to Probability (Equally Likely Model)

$$P(A) = \frac{s}{n} = \frac{\text{number of ways A can occur}}{\text{number of different simple events}}$$

Slide 6

Rule 3: Subjective Probabilities

***P*(*A*), the probability of event *A*, is found by simply guessing or estimating its value based on knowledge of the relevant circumstances.**

This is not reliable. It is used only when experts are involved to do the estimation.

Law of Large Numbers

Slide 7

As an experiment is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.

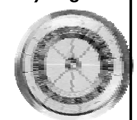
Example

Slide 8

Roulette You plan to bet on number 13 on the next spin of a roulette wheel. What is the probability that you will lose?

Solution A roulette wheel has 38 different slots, only one of which is the number 13. A roulette wheel is designed so that the 38 slots are equally likely. Among these 38 slots, there are 37 that result in a loss. Because the sample space includes equally likely outcomes, we use the classical approach (Rule 2) to get

$$P(\text{loss}) = \frac{37}{38}$$

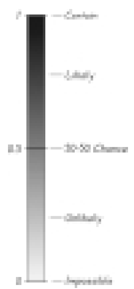


Possible Values for Probabilities

Slide 9

Can NOT be bigger than 1

Can NOT be negative!



Definition

Slide 10

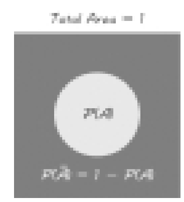
The complement of event *A*, denoted by $\bar{A}$, consists of all outcomes in which the event *A* does *not* occur.

Example: Roll a die.

Sample Space: S={1,2,3,4,5,6}

Let an event: A={1,2}

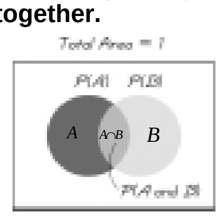
Then $\bar{A}$={3,4,5,6}



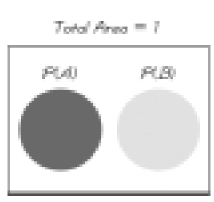
Venn Diagram

Slide 11

Events *A* and *B* are disjoint (or mutually exclusive) if they cannot both occur together.



Intersects



disjoint

***A* and $\bar{A}$ are mutually exclusive.**

Example

Slide 12

Birth Genders In reality, more boys are born than girls. In one typical group, there are 205 newborn babies, 105 of whom are boys. If one baby is randomly selected from the group, what is the probability that the baby is *not* a boy?

Solution Because 105 of the 205 babies are boys, it follows that 100 of them are girls, so

$$P(\text{not selecting a boy}) = P(\overline{\text{boy}}) = P(\text{girl}) = \frac{100}{205} = 0.488$$

$$\text{or} \qquad = 1 - P(\text{boy}) = 1 - \frac{105}{205} = 0.488$$

Definitions

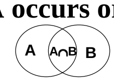
- **The actual odds against event *A* occurring are the ratio $P(\bar{A})/P(A)$, usually expressed in the form of *a:b* (or "*a* to *b*"), where *a* and *b* are integers having no common factors.**
- **The actual odds in favor event *A* occurring are the reciprocal of the actual odds against the event.**
- **The payoff odds against event A = (net profit) : (amount bet)**

3.3 Additive Rule

Compound Events

Let A and B be two events

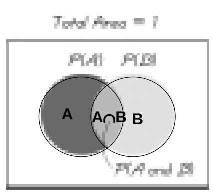
Union A∪B: A occurs or B occurs or they both occur.



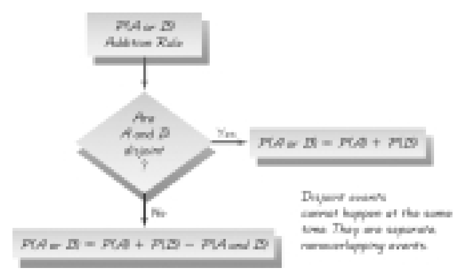
Intersection A∩B: Both A and B occur.

Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Applying the Addition Rule



Example

	Men	Women	Boys	Girls	Totals
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	56	2223

Find the probability of randomly selecting a man or a boy.

$$P(\text{man or boy}) = \frac{1692}{2223} + \frac{64}{2223} = \frac{1756}{2223} = 0.79$$

* Disjoint *

Example

	Men	Women	Boys	Girls	Totals
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	45	2223

Find the probability of randomly selecting a man or someone who survived. *** NOT Disjoint ***

$$P(\text{man or survivor}) = \frac{1692}{2223} + \frac{706}{2223} - \frac{332}{2223} = \frac{2066}{2223} = 0.929$$