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Gehring's lemma for nondoubling measures.

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Let $Q_0 \subset \mathbf{R}^n$ be a fixed cube with sides parallel to the coordinate axes, $w > 0$ a weight on Q_0, and $1 < p < \infty$. A positive function $g \in L^p_w(Q_0)$ is said to satisfy a reverse Hölder inequality if there exists some $C \geq 1$ such that, for all cubes $Q \subset Q_0$ with sides parallel to the coordinate axes,

$$\Big(w(Q)^{-1}\int_Q g(x)^p w(x)dx\Big)^{1/p} \leq Cw(Q)^{-1}\int_Q g(x)w(x)dx.$$

Then one writes $g \in RH_p(w)$. A measure μ on $\mathbf{R}^n$ is doubling if $\mu(B(x,2r)) \leq C\mu(B(x,r))$ for all $x \in \operatorname{supp}(\mu)$, $r > 0$. Suppose that the measure $\mu := w(x)dx$ is doubling and $g \in RH_p(w)$. Then, by Gehring's lemma [F. W. Gehring, Acta Math. **130** (1973), 265–277; MR0402038 (53 #5861)], there exists some $\varepsilon > 0$ such that $g \in RH_{p+\varepsilon}(w)$. In the paper under consideration the authors extend this result to the case where $\mu = w(x)dx$ is not assumed to be doubling. The authors also prove a more general version of this self-improvement result which involves two different functions g and h.

The proofs involve ideas from real interpolation theory and covering lemmas. In particular, the authors use a characterization of the K-functional of the pair $(L^1_w(\mathbf{R}^n), L^\infty(\mathbf{R}^n))$ in terms of a maximal operator found by I. U. Asekritova et al. [Studia Math. **124** (1997), no. 2, 107–132; MR1447618 (98g:46032)], and a modification of the usual Calderón-Zygmund decomposition suitable for nondoubling measures obtained by J. Mateu et al. [Duke Math. J. **102** (2000), no. 3, 533–565; MR1756109 (2001e:26019)]. *Xavier Tolsa* (E-BARA)

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