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Weighted norm inequalities and indices. (English. English summary)

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Consider the generalized Hardy operators $P_{1-\lambda}^{(q)}$ for $0 < q < \infty$ and $0 \leq \lambda < 1$ defined as

$$P_{1-\lambda}^{(q)} f(t) = \left(\frac{1}{t^{1-\lambda}} \int_0^t |f(x)|^q \frac{dx}{x^\lambda} \right)^{1/q},$$

and their adjoints. These operators control the rearrangement inequalities of several classical operators, which makes the study of their boundedness properties when acting on decreasing functions interesting. Sometimes it is enough to test the operator on the class of characteristic functions of the intervals $(0, r)$ for all $r > 0$, that is, on the decreasing characteristic functions in $(0, \infty)$. A quasilinear operator T is $(\chi - X)$-bounded on the quasi-Banach space X if $\|Tf\|_X \leq c\|f\|_X$ for all f in such a class.

In this paper the generalized Hardy operators are considered in connection with the theory of Boyd and Zippin indices. A characterization obtained for the Zippin upper index of a quasi-Banach function space X is the following: $P_{1-\lambda}^{(q)} \circ P_{1-\lambda}^{(q)}$ is $(\chi - X)$-bounded if and only if $\overline{z}_X < (1 - \lambda)/q$. A similar characterization for the Zippin lower index involves the adjoint operators. The characterization does not hold with $P_{1-\lambda}^{(q)}$, so the use of iterates is crucial here.

Using the Zippin indices the authors obtain a characterization of the classes of weights for which the generalized Hardy operators and their adjoints are bounded on weighted classical and weak Lorentz spaces. Thus they give a unified approach which contains as particular cases results due to several authors. *Javier Duoandikoetxea* (Bilbao)