Solutions to Quiz # 2 (STA 4443)

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from the geometric distribution with p.m.f.
$f(x; p) = (1 - p)^{x-1}p, \; x = 1, 2, 3, \cdots$. Find the maximum likelihood estimator of $p$.

**Solution.** The likelihood function is given by

$$L(p) = (1 - p)^{x_1-1} \cdots (1 - p)^{x_n-1}p = p^n(1 - p)^{\sum_{i=1}^n x_i - n}.$$  

The logarithm of the likelihood function is

$$\ln L(p) = n \ln p + \left( \sum_{i=1}^n x_i - n \right) \ln(1 - p).$$

Setting the first derivative equal to zero and solving for $p$ yields

$$\frac{d \ln L(p)}{dp} = \frac{n}{p} - \frac{\sum_{i=1}^n x_i - n}{1 - p} = 0$$

$$p = \frac{n}{\sum_{i=1}^n x_i} = \frac{1}{\bar{x}}.$$  

Thus, the maximum likelihood estimator of $p$ is

$$\hat{p} = \frac{n}{\sum_{i=1}^n X_i} = \frac{1}{\bar{X}}.$$  

2. A random sample of size 16 from $N(\mu, 64)$ yielded $\bar{x} = 80$. Find the following confidence intervals for $\mu$:

(a) 99%. (b) 95%. (c) 90%. (d) 70%.

**Solution.** (a) Note that $\alpha = 0.01$ and $z_{\alpha/2} = z_{0.005} = 2.576$. So the desired 99% confidence interval is

$$[\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}] = [80 - (2.576)(\sqrt{64}/\sqrt{16}), 80 + (2.576)(\sqrt{64}/\sqrt{16})]$$

$$= [74.848, 85.152].$$

Similarly, we can find the confidence intervals in (b), (c) and (d):

(b) $[76.08, 83.92]$ (with $\alpha = 0.05$, $z_{\alpha/2} = 1.96$);

(c) $[76.71, 83.29]$ (with $\alpha = 0.10$, $z_{\alpha/2} = 1.645$);

(d) $[77.928, 82.072]$ (with $\alpha = 0.30$, $z_{\alpha/2} = 1.036$).