Solutions to Assignment 5 (STA 4443)

7.2-2. (a) For the left-sided test, the test statistics is

\[ Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{\bar{X} - 13}{0.2 / \sqrt{25}}. \]

The critical region is

\[ z = \frac{\bar{x} - 13}{0.2 / \sqrt{25}} \leq -z_\alpha = -z_{0.025} = -1.96. \]

(b) Since \( z = (12.9 - 13)/(0.2 / \sqrt{25}) = -2.5 < -1.96 \), we reject \( H_0 \).
(c) The \( p \)-value = \( P(Z \leq z) = P(Z \leq -2.5) = P(Z \geq 2.5) = 0.0062 \).

7.2-4. (a) The test statistics is

\[ T = \frac{\bar{X} - 7.5}{S / \sqrt{10}} \]

and the critical region is \(|t| \geq t_{\alpha/2}(n-1) = t_{0.025}(9) = 2.262\). The figure is omitted.
(b) Based on the given data, we find that \( \bar{x} = 7.55 \) and \( s = 0.1027 \). Since

\[ |t| = \frac{|7.55 - 7.5|}{0.1027 / \sqrt{10}} = 1.54 < 2.262, \]

we do not reject \( H_0 \).
(c) A 95% confidence interval for \( \mu \) is

\[ [7.55 - 2.262(0.1027)/\sqrt{10}, 7.55 + 2.262(0.1027)/\sqrt{10}] = [7.48, 7.62]. \]

Hence, \( \mu = 7.5 \) is contained in this interval. This result is consistent with the conclusion obtained in part (b).

7.2-6. (a) \( H_0 : \mu = 3.4; \)
(b) \( H_1 : \mu > 3.4; \)
(c) The test statistics is \( T = (\bar{X} - 3.4)/(S/3) \).
(d) The critical region is \( t \geq t_\alpha(n-1) = t_{0.05}(8) = 1.860 \). The figure is omitted.
(e) From the given data, we find \( \bar{x} = 3.556 \) and \( s = 0.167 \). So, the value of the test statistics is \( t = (3.556 - 3.4)/(0.167/3) = 2.802 \).
(f) Since $t = 2.802 > 1.860$, we reject $H_0$.

(g) Since $2.306 < 2.802 < 2.896$, we have $0.01 < p-value < 0.025$. The true $p$-value is 0.0116.

**7.2-14.** The critical region is

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{17}} \geq t_{0.05}(16) = 1.746.$$ 

Since $\bar{d} = 4.765$ and $s_d = 9.087$, we have $t = 2.162 > 1.746$ and consequently we reject $H_0$.

**7.3-2.** (a) The test statistics is

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{15S^2_X + 12S^2_Y}{27} \left( \frac{1}{16} + \frac{1}{13} \right)}}$$

and the critical region is $t \geq t_{0.01}(27) = 2.473$.

(b) The value of the test statistics is

$$t = \frac{415.16 - 347.40}{\sqrt{\frac{15(1356.75) + 12(692.21)}{27} \left( \frac{1}{16} + \frac{1}{13} \right)}} = 5.570.$$ 

Since $t = 5.570 > 2.473$, we reject $H_0$.

**7.3-4.** (a) The test statistics is

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{12S^2_X + 15S^2_Y}{27} \left( \frac{1}{13} + \frac{1}{16} \right)}}$$

and the critical region is $t \leq -t_{0.05}(27) = -1.703$.

(b) The value of the test statistics is

$$t = \frac{72.9 - 81.7}{\sqrt{\frac{12(25.6^2) + 15(28.3^2)}{27} \left( \frac{1}{13} + \frac{1}{16} \right)}} = -0.869.$$ 

Since $t = -0.869 > -1.703$, we do not reject $H_0$.

(c) Note that for $r = 27$, we have $t = 0.869 \in (0.684, 1.314)$. We find that $0.10 < p-value < 0.25$. 

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7.3-6. (a) Assuming $\sigma^2_X = \sigma^2_Y$, the test statistic is

$$ T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{9\sigma^2_X + 9\sigma^2_Y}{18} \sqrt{\frac{1}{10} + \frac{1}{10}}}}. $$

The critical region is

$$ |t| = \frac{|\bar{x} - \bar{y}|}{\sqrt{\frac{9\sigma^2_X + 9\sigma^2_Y}{18} \sqrt{\frac{1}{10} + \frac{1}{10}}}} \geq t_{0.05/2}(20-2) = 2.101. $$

(b) Based on the given sample data, we find $|t| = | - 2.151| > 2.101$; therefore we reject $H_0$.

(c) The $p$-value is $2P(T > |t|)$ which is between 0.02 and 0.05.

(d) The box plots are omitted.

7.3-10. Based on the summary data, we have

$$ t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{24s^2_x + 28s^2_y}{52} \left( \frac{1}{25} + \frac{1}{29} \right)}} = 3.381 > 2.326 = t_{0.01}(52)(\approx z_{0.01}). $$

So, we reject $H_0 : \mu_X = \mu_Y$.

7.4-4. (a) The critical region is

$$ \chi^2 = \frac{19s^2}{(0.095)^2} \leq \chi^2_{1-\alpha}(n-1) = \chi^2_{0.05}(19) = 10.12. $$

The observed value of the test statistics is

$$ \chi^2 = \frac{19(0.065)^2}{(0.095)^2} = 8.895, $$

which is less than 10.12, so we reject $H_0$ and conclude that the company was successful.

(b) Since $\chi^2_{0.075}(19) = 8.907$, we see that the $p$-value is approximately 1-0.075=0.025.

7.4-8. The critical region is $s^2_x/s^2_y \geq F_\alpha(n-1, m-1) = F_{0.05}(12, 8) = 3.28$. Based on the summary data, we find $s^2_x/s^2_y = 9.88/4.08 = 2.42 < 3.28$ and therefore we do not reject $H_0$. 

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