Impulse control with random reaction periods: A central bank intervention problem

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ABSTRACT

We model an impulse control problem when the controller’s action affects the state as well as the dynamics of the state process for a random amount of time. We apply our model to solve a central bank intervention problem in the foreign exchange market when the market observes and reacts to the bank’s interventions.

1. Introduction

The exchange rate of a country’s currency (the number of domestic currency units per unit of foreign currency or a given basket of foreign currencies) has always been recognized to play a central role in the import and export operations of the country in particular and its economy in general. A relatively strong exchange rate can hurt an exporter as it weakens its ability to compete in the world market. To prevent that from happening after the earthquake and tsunami on March 11, 2011 in Japan, the central bank of Japan intervened to weaken the yen as it had reached its strongest value since World War II [25]. This intervention was coordinated with the G7 countries to help Japan in its export market according to the BBC [39]. On the other hand, a relatively weak exchange rate can hurt an importer by making imported goods more expensive. Consequently, it is important for a country to keep its exchange rate within a band or close to a target rate, determined by the country’s central bank.

Many intervention options are available to a central bank, viz. intervention against depreciation [23], intervention against appreciation [38], coordinated intervention [16], unilateral intervention [41], announced intervention, secret intervention [4], spot market intervention [9], forward market intervention, options market intervention [37] etc. For example, if a central bank chooses to intervene against appreciation (depreciation), then it can buy (sell) foreign currencies in the exchange rate market.

A central bank faces the problem of finding the best possible intervention policy for its currency, referred to as the CBI problem hereafter. Cadenillas and Zapatero [12,13], and Mundaca and Øksendal [35] model the CBI problem as an impulse control problem under the assumption that the market does not observe the interventions of the central bank. But, in practice, the foreign exchange market observes and reacts to such interventions. This may affect the dynamics of the exchange rate process temporarily, and was the case in the Japanese example mentioned above. The market reactions can either increase or decrease the exchange rate volatility depending on various factors and this has been already documented in [2,3,10,14,17,24,33,34,42]. Yet, the basic impulse control models in [12,13,35] to treat the CBI problem do not consider the market to observe and react to the interventions resulting in an altered dynamics of the exchange rate.

In this paper, we propose an impulse control model that allows for more general problems than the CBI problem with market reactions. The model involves a reaction-adjusted cost operator in the formulation of the resulting quasi-variational inequality (QVI). This represents an extension of the basic QVIs studied in [6].

The rest of the paper is organized as follows. We introduce the impulse control model with random reaction periods in Section 2. We apply it to analytically solve a CBI problem with market reactions in Section 3. Section 4 concludes the paper.
2. Impulse control model with random reaction periods

Let \((\Omega, \mathcal{F}, P)\) be a probability space equipped with a filtration \(\{\mathcal{F}_t\}_{t \geq 0}\) generated by a 1-dimensional Brownian motion \(B(t)\). Consider the diffusion process of the form

\[
dX(t) = \mu(X(t))dt + \sigma(X(t))dB(t), \quad X(0) = x,
\]

where \(\mu\) and \(\sigma\) are real-valued functions on \(\mathbb{R}\) satisfying some conditions so that there is a unique solution of (1) (see [30]).

We consider \(n\) different types of interventions referred to as type 1, type 2, ..., type \(n\). Specifically, when the system is in state \(y\) at time \(t\) with the dynamics (1), and an impulse \((i, \zeta) \in \{1, 2, \ldots, n\} \times \mathbb{Z}^i \subset \mathbb{N} \times \mathbb{R}\) of type \(i\) and size \(\zeta\), where \(\mathbb{Z}^i\) is the set of admissible impulse values associated with the intervention of type \(i\), is applied, the state jumps immediately from \(X(t^-) = y\) to \(X(t) = I_j(y, \zeta)\), where \(I_j\) is a given function. Furthermore, the system dynamics switches to

\[
dx_i(t) = \mu_i(X_i(t))dt + \sigma_i(X_i(t))dR_i(t),
\]

with \(\mu_i\) and \(\sigma_i\) defined in ways similar to (1), and it persists for a random period of duration \(T_i^j\). In the course of the infinite horizon, a sequence of such interventions will be made. Let \(\tau_i\) and \(\nu_i\) denote the time and the type of the \(j\)th intervention, which will persist until time \(\tau_j + T^j_j\), where \(T^j_j \geq 0\) is a bounded random variable for \(j \in \{1, 2, \ldots, n\}\), \(j = 1, 2, \ldots\). We call the period \((\tau_j, \tau_j + T^j_j)\), the reaction period. The system reverts back to the pre-intervention process at the end of the reaction period. We assume that \(T^j_j \sim T_i\) for all \(j\), where \(T_i\) has the distribution function \(F_i, i = 1, 2, \ldots, n\), and \(T_i\)'s are i.i.d. and independent of the process \(B(t)\). We now provide the definition of an impulse control.

**Definition 1.** An impulse control is a double sequence \(\nu = (\tau_1, \tau_2, \ldots, (i_1, \zeta_1), (i_2, \zeta_2), \ldots, (i_r, \zeta_r), \ldots)\) to the state process \(X(t)\) with \(X(0) = x\), the corresponding interrupted state process \(X^{(\nu)}(t)\) can be defined by

\[
X^{(\nu)}(t) = X_i(t), \quad 0 \leq t < \tau_1,
\]

\[
X^{(\nu)}(\tau_1) = I_{i_1}(X^{(\nu)}(\tau_1 -), \zeta_1); \quad j = 1, 2, \ldots,
\]

\[
X^{(\nu)}(\tau_1 + T^j_1) = X^{(\nu)}(\tau_1 + T^j_1) = \left\{ \begin{array}{ll}
X^{(\nu)}(\tau_1 + T^j_1) & \text{for } \tau_j + T^j_j < \tau_j + T^j_{j+1}; \\
X^{(\nu)}(\tau_j + T^j_j) & \text{for } \tau_j + T^j_j < \tau_j + T^j_{j+1};
\end{array} \right.
\]

\[
= \ldots
,j = 1, 2, \ldots.
\]

Now let \(f(x) : \mathbb{R} \to [1, 2, \ldots, n]\) be the running cost and \(K_0 : \mathbb{Z}^i \to \mathbb{R}\) be the cost of type \(i\) intervention. Then we can define the performance criterion by

\[
J^{(\nu)}(x) := \mathbb{E}\left[\int_0^\infty e^{-rt}f(X^{(\nu)}(t))dt + \sum_{j=1}^\infty e^{-rT_j}K_0(\zeta_j)\right],
\]

where \(r > 0\) is the discount rate. Therefore, we are interested in minimizing the functional given in (7) over the set of all admissible impulse controls defined below:

**Definition 2.** \(\mathcal{V}\) is the set of admissible impulse controls, if for all \(x \in \mathbb{R}, \nu \in \mathcal{V}\),

(i) \(\nu\) is a unique solution \(X^{(\nu)}(t), t \geq 0\), of the system (3)–(6) exists,

(ii) \(\lim_{t \to \infty} \tau_j = \infty\) a.s.,

(iii) \(E\left[\int_0^\infty e^{-rt}f(X^{(\nu)}(t))dt + \sum_{j=1}^\infty e^{-rT_j}K_0(\zeta_j)\right] < \infty,\)

(iv) \(E\left[\sum_{j=1}^\infty e^{-rT_j}K_0(\zeta_j)\right] < \infty,\)

(v) \(\tau_{j+1} = \tau_j + T^j_j, j = 1, 2, \ldots\).

Note that condition (v) means that no interventions are possible during a reaction period. We denote the value function of our impulse control problem by \(\phi\). That is, for all \(x \in \mathbb{R}\),

\[
\phi(x) = \inf_{\nu \in \mathcal{V}} \mathbb{E}\left[\int_0^\infty e^{-rt}f(X^{(\nu)}(t))dt + \sum_{j=1}^\infty e^{-rT_j}K_0(\zeta_j)\right],
\]

Hence, our objective is to find the function \(\phi(x)\) and \(\nu^* \in \mathcal{V}\).

2.1. Quasi variational inequality (QVI)

We present now the corresponding QVI of the impulse control problem to the random reaction periods. We first introduce some preliminary definitions. Let the infinitesimal generators \(A\) and \(A_i\) be defined as follows:

\[
A\phi(x) = \mathbb{E}(x) + \frac{1}{2}a^2(x, \phi_{xx}) - r \phi(x),
\]

\[
A_i\phi(x) = \mathbb{E}(x) + \frac{1}{2}a^2(x, \phi_{xx}) - r \phi(x) \quad \text{for } i = 1, 2, \ldots, n.
\]

Now, let \(i \in \{1, 2, \ldots, n\}\) and consider the Cauchy problem

\[
\frac{\partial \phi_i(t, x)}{\partial t} = A_i \phi_i + f: \quad \text{with } \phi_i(0, x) = \psi(x).
\]

We denote the solution of (8) by \(\phi(t, x; \psi)\) and define the minimum cost operator \(M^i\) for an intervention of type \(i\) by

\[
M^i\phi(x) = \inf_{\nu \in \mathcal{V}} \mathbb{E}\left[\int_0^\infty e^{-rt}f(X^{(\nu)}(t))dt + \sum_{j=1}^\infty e^{-rT_j}K_0(\zeta_j)\right],
\]

We now define the minimum cost operator \(M_i\) for the impulse control model with random reaction periods and the corresponding QVI. The derivation of the QVI is standard and the reader is referred to [6] for details.

**Definition 3.** \(M_i\phi(x) = \min \{M^i\phi(x); i \in \{1, 2, \ldots, n\}\}.

2.4. Let \(\phi\) be a \(C^1\) function. Then we say that \(\phi\) satisfies the QVI for impulse control with random reaction periods, if for all \(x \in \mathbb{R}\), we have

\[
\phi(x) \leq M_i\phi(x),
\]

\[
A\phi(x) + f(x) \geq 0,
\]

\[
[A\phi(x) + f(x)](\phi(x) - M_i\phi(x)) = 0.
\]

A solution of the above QVI separates the real line into two disjoint regions. Namely, a continuation region

\[
\mathcal{C} = \{x: \phi(x) < M_i\phi(x) \text{ and } A\phi(x) + f(x) = 0\},
\]

and an intervention region

\[
\mathcal{I} = \{x: \phi(x) = M_i\phi(x) \text{ and } A\phi(x) + f(x) > 0\}.
\]

The intervention region \(\mathcal{I}\) consists of \(n\) sub-regions determined by \(M_i\)'s for \(i = 1, 2, \ldots, n\), and some of these sub-regions may be empty. We also recall that condition (v) of Definition 2 does not allow us to intervene during the random reaction periods. Hence, we control the process only when it reaches the boundary of the continuation region given that the process is in the pre-intervention regime given by (1).

In the next section, we apply the QVI to solve a CBI problem in the foreign exchange market when an intervention affects the market dynamics for a random period of time.
3. Central bank intervention (CBI) problem

3.1. A brief literature review

Many countries used target-zone models to derive their central bank intervention policies until the late 90’s. In a target-zone regime, the exchange rate of a country’s currency is allowed to move only within a specified band in the sense that the country’s central bank intervenes to prevent the exchange rate from moving outside the band. In [28], Krugman introduced the standard target-zone model, which has been discussed extensively in references such as [7,19–21,29,32,40]. Jeanblanc-Picquet [26] initiated the application of the stochastic impulse control to this problem, which was later extended by Korn [27]. In all these models, the band for the exchange rate is given exogenously and no intervention takes place when the exchange rate is strictly inside the band. However, it is often observed that most of the interventions in the foreign exchange rate market occur within the band (see [1,8] for empirical evidence), because the cost of keeping the exchange rate within the band increases due to speculative attacks at the boundary of the band. Furthermore, it is also not unusual to observe interventions in the foreign exchange rate market when the exchange rate drifts too far away from a given benchmark; see [5,18]. All this means that the target-zone models are not suitable in practice.

Mundaca and Øksendal [35] provided an improved model which eliminates the drawbacks of the target-zone models using the impulse control theory. They showed that it is not necessary to set a target band exogenously. Rather, the correct target band can be derived endogenously as part of the solution to the optimization problem, using a cost which is the sum of the cost of intervention and the running cost given by an increasing function of the distance between the target rate and the current rate. They used a standard Brownian motion model for their underlying exchange rate, but did not provide an exact analytic solution to the problem. Cadenillas and Zapatero [12,13] provided an explicit solution to this problem when the underlying exchange rate follows a geometric Brownian motion. Øksendal and Sulem [31] argued for the use of the stochastic impulse control theory in a very broad setting using a jump diffusion process. They also discussed many different intervention problems when the underlying process follows a jump diffusion process.

Domínguez [17] empirically studied the effect of foreign exchange interventions by central banks on the behavior of exchange rates, and showed that the intervention operations that followed the Plaza Agreement generally increase exchange rate volatility. Moreover, [2,10,14] showed that the central bank interventions often increase the exchange rate volatility temporarily after an intervention. In contrast, Beine et al. [3] and Mundaca [33] explained that central bank interventions can have a stabilizing effect by reducing exchange rate volatility. One take away from this literature is that market reactions to the central bank interventions influence the exchange rate dynamics, and should therefore be further studied.

3.2. CBI problem with market reactions

We treat the CBI problem with market reactions using the model developed in Section 2. We assume that the market observes and then reacts to the interventions by the central bank in the foreign exchange market. In particular, we assume that the process driving the exchange rate dynamics is affected by intervention for a random length of time, and then reverts back to the pre-intervention process.

We focus on the problem of finding the optimal intervention policy which should be implemented by the central bank of a country that wants to keep its exchange rate (against a given basket of foreign currencies) within a band or close to a target rate. There are both fixed and proportional costs associated to each intervention as considered in [12,35]. The proportional cost mainly consists of transaction costs in this case. Each intervention is a large operation involving several executives, multiple traders, and many hours of discussions, and hence the fixed cost is not negligible in the CBI problem. We use a geometric Brownian motion $X(t)$ to model the exchange rate at time $t$ when there are no interventions. Thus,

$$\text{d}X(t) = \mu X(t)\text{d}t + \sigma X(t)\text{d}B(t), \quad X(0) = x. \quad (9)$$

Here $\mu = r - r_f$ and $\sigma > 0$ are constants, where $r$ is the domestic risk-free interest rate, $r_f$ is the foreign risk-free interest rate, and $\sigma$ is the exchange rate’s volatility (see [22] and references therein).

Assume that the central bank is allowed to perform two types of interventions, namely, intervention to devalue the domestic currency (type 1) and intervention to increase the value of the domestic currency (type 2). Let $\Gamma_1(x, \xi) = x + \xi$, $\Gamma_2(x, \xi) = x - \xi$ and $\mathcal{Z} = \mathbb{Z}^2 = [0, \infty)$. Consider that at time $t_j$, $j = 1, 2, \ldots$, the central bank’s $j$th intervention is of type $i \in \{1, 2\}$. Then, we assume that the dynamics switches to

$$\text{d}X^i(t) = \mu X^i(t)\text{d}t + \sigma X^i(t)\text{d}B(t), \quad t_j \leq t \leq t_{j+1} \quad (10)$$

where $T_j \geq 0$ is a bounded random variable. Note that the process $X^i$ in (10) has the same drift as in (9) but a different volatility $\sigma_i$. Furthermore, $X^i(t_j)$ equals the latest pre-intervention exchange rate plus $\xi$ (minus $\xi$) for type 1 (type 2) intervention, respectively. Market reactions end at $t_j + T_j$ and the exchange rate process reverts back to the pre-intervention dynamics at that time. The assumptions on $T_j$ are as in Section 2 and the intervened process $X^i(t)$ is defined by the system (3)–(6) with (9) and (10) replacing (1) and (2), respectively.

We define the running cost incurred by deviating from the target rate $\rho$ by $f(x) = (x - \rho)^2$, and the cost of making an intervention of type 1 or type 2 by $K_i(x, \xi, \zeta) = C + c_\zeta$ or $K_i(x, \xi, \zeta) = D + d_\zeta$, respectively. Here $C, D$ are the fixed costs, and $c$ and $d$ are the proportional costs of an intervention of type 1 and type 2, respectively.

The central bank wants to determine the optimal impulse control that minimizes the total expected discounted future costs. Specifically, the objective is to find an admissible impulse control that minimizes the functional $J^{(1)}$ defined by

$$J^{(1)}(x) = E \left[ \int_0^\infty e^{-r\tau} f(X^i(t))\text{d}t + \sum_{i=1}^\infty e^{-r\tau} K_i(x(\zeta)) \right],$$

where $r$ is the discount rate. We denote the value function as $\Phi(x) = \inf \{ J^{(1)}(x); \ a \in \mathcal{V} \} = J^{(1)*}(x), \ x \in \mathbb{R}$ and we look for $\Phi^* \in \mathcal{V}$ for each $x$.

3.3. Solution of the QVI

We now solve the QVI that we derived in Section 2 for the CBI problem. We first note that the infinitesimal generators $A$ and $A\Phi$, and the minimum cost operator $M \Phi$, are reduced to the following forms in this case:

$$A \Phi(x) = \mu x \frac{\partial \Phi}{\partial x} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 \Phi}{\partial x^2}, \quad A \Phi(x) = \mu x \frac{\partial \Phi}{\partial x} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 \Phi}{\partial x^2}, \quad r \Phi(x), \quad (11)$$

$$M \Phi(x) = \min \{ M^1 \Phi(x), M^2 \Phi(x) \}, \quad (12)$$

$$M^1 \Phi(x) = \inf \{ C + c \xi + \Gamma_1(x + \xi); \ \ \xi > 0 \} \quad (11)$$

$$M^2 \Phi(x) = \inf \{ D + d \xi + \Gamma_2(x - \xi); \ \ \xi > 0 \} \quad (12)$$

where $\Gamma_i(m) = \int_0^\infty e^{-\gamma t} \xi(t, m; \Phi)\text{d}F_i(t)$ for $i \in \{1, 2\}$.
Assume that there exists an optimal policy for a given exchange rate $x$ and the central bank follows the optimal policy starting with this rate $x$. The optimal cost associated with this policy is $\Phi(x)$. An alternate policy is as follows: the central bank starts with the exchange rate $x$ and selects the best immediate intervention, then follows an optimal policy. The cost associated with the alternate policy is $M_t \Phi(x)$. Since the first policy is optimal, we always have $M_t \Phi(x) \geq \Phi(x)$, and it is optimal to intervene whenever $M_t \Phi(x) = \Phi(x)$. This observation is consistent with the definition of the QVI in Section 2.

We now conjecture that the solution of the QVI for the CBI problem is a two-band control (or a simple policy as termed in [15]) characterized by four parameters $L$, $l$, $u$, and $v$ such that $0 < L < l < u < v < \infty$, and

\[ \forall x \in (0, L], \quad \Phi(x) = C + c(l - x) + T_1(l), \]
\[ \forall x \in [L, u], \quad \Phi(x) = D + d(x - u) + T_2(u), \]
\[ \forall x \in (u, v], \quad 0 = A \Phi(x) + f(x). \]

The optimal policy is

\[ \Phi^*(x) = \begin{cases} c, & \text{if } x < L, \\ d, & \text{if } x \geq L, \end{cases} \]

and a two-band impulse control indirectly by

\[ \hat{t}_j = \inf \{ t > \hat{t}_j + T_j^b : X_j(t) \not\in (L, U) \}, \quad \text{with } \hat{t}_0 = 0, \]

\[ X_j(t) = X_j(0) + \hat{\zeta}(t) = \begin{cases} x, & \text{if } x \leq L, \\ x + T_j(u) = \frac{1}{2} \left( x - \frac{c}{\sigma^2} \right)^2 + 2 \rho \sigma \tilde{\chi} + \rho^2 \tilde{\chi}, \end{cases} \]

\[ \text{and } A = \begin{cases} 1, & \text{if } x < L, \\ 0, & \text{if } x \geq L, \end{cases} \]

where $X_j(0)$ is the result of applying $\hat{\zeta} = (\hat{t}_1, \ldots, \hat{t}_j, \hat{\zeta}_1, \ldots, \hat{\zeta}_j)$. If we let $\tilde{\nu} = (\tilde{t}_1, \ldots, \tilde{t}_j, \tilde{\zeta}_1, \ldots, \tilde{\zeta}_j)$, then we can use a standard verification argument to show that $J^*(x) = \Phi(x)$ for all $x \in [0, \infty)$. We now summarize the QVI characteristics associated with our two-band impulse control problem as follows:

Table 1

<table>
<thead>
<tr>
<th>Without market reactions</th>
<th>L</th>
<th>U</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.584</td>
<td>1.155</td>
<td>1.270</td>
<td>2.315</td>
</tr>
<tr>
<td>With market reactions</td>
<td>0.530</td>
<td>1.101</td>
<td>1.208</td>
</tr>
</tbody>
</table>

We provide the conditions under which a solution of the two-band control problem solves the QVI in the following proposition.

**Proposition 1.** Suppose that $\Phi \in C^1$ is a solution of (14)–(20) such that

\[ L < \frac{1}{2} \left( c(\mu - r) + 2 \rho G_1(c, C, l) \right), \]
\[ U > \frac{1}{2} \left( d(\mu - r) + 2 \rho G_2(-d, d, u) \right), \]

\[ \Phi(x) < C - c x + \inf_{x < u} \left[ c y + T_1(y) \right], \quad \forall x \in (L, U), \]

\[ \Phi(x) < D + d x + \inf_{L < x < \infty} \left[ -d y + T_2(y) \right], \quad \forall x \in (L, U), \]

where $G_1(\alpha, \beta, \theta) = \left( [\alpha(\mu - r) + 2 \rho^2] - 4 \left( \rho^2 - r T_1(\theta) - r \beta - r \alpha \theta \theta \right) \right)^{\frac{1}{2}}$. Then it is a solution of the QVI for the CBI problem.

**Proof.** We first note that $A \Phi(x) + f(x) = \left( -c \mu x - r \left( C + c(l - x) + T_1(l) \right) + (x - \rho^2) \right)$, if $0 < x < L$, and

\[ A \Phi(x) + f(x) = \left( -c \mu x - r \left( D + d(x - u) + T_2(u) \right) \right), \quad \text{if } L < x < \infty. \]

Clearly, $A \Phi(x) + f(x) = A \Phi(x) + f(x) = 0$ in $(L, U)$. By conditions (21) and (22), we have that $A \Phi(x) + f(x) > 0$ in $(0, L]$ and $A \Phi(x) + f(x) > 0$ in $(L, \infty)$. Moreover, $A \Phi(x) + f(x) > 0$ in $(0, L] \cup [U, \infty)$, respectively.

We now observe that $M_t \Phi(x) = C + c(l - x) + T_1(l)$ in $(0, L]$ and $M_t \Phi(x) = D + d(x - u) + T_2(u)$ in $(U, \infty)$. Hence, $M_t \Phi(x) = \Phi(x)$ in $(0, L] \cup [U, \infty)$. Finally, conditions (23) and (24) guarantee that $\Phi(x) < M_t \Phi(x)$ in $(L, U)$. Hence, $\Phi$ is a solution of the QVI.

We now provide a numerical example.

**3.4. Numerical example and comparison**

We need to solve the system (15)–(20) of six equations with six unknowns to find the solution of the CBI problem. However, it is not possible to find an explicit solution to the system. So, we solve it numerically using Newton's method. We assume that $\sigma_1 = \sigma_2$ and, $T_1^j \sim T_2$ and $T_2 \sim T_2$, $j = 1, 2, \ldots$ for simplicity.

Based on the empirical observations in [17], we first assume that the exchange rate volatility increases during the reaction period. Hence, we use the following model parameters: $r = 0.06, \mu = 0.1, \sigma = 0.3, \sigma_1 = 0.4, \rho = 1.4, C = D = 0.5, c = d = 0.2$, and $T \sim \text{Exponential}(2)$. We compute the optimal parameters of the two-band control using computer programs in Matlab and "R". We compare our optimal parameters and the value function with the case without market reactions (see [12]) in Table 1 and Fig. 1.

From Table 1, we observe that the optimal bands are wider in the case with market reactions compared to the case without market reactions. This means that the central bank would intervene less frequently if the market reactions increase the exchange rate volatility. We also observe that the optimal policy is more expensive than its corresponding value without the market reactions in this case (see Fig. 1).

Beine et al. [3] showed that a central bank intervention can have a stabilizing effect on exchange rate volatility. This means that the exchange rate volatility may decrease temporarily after
problem (see [36]) with market reactions. Constantinides and Richard [15] used the impulse control theory to study a cash management problem of a generic firm. However, whenever a mutual fund manager deposits (withdraws) a large amount of cash into (from) his cash inventory, the investors in the mutual fund observe the managers actions and react to that. These reactions change deposit and withdrawal patterns of the investors and hence change the dynamics of the demand process of the mutual fund’s cash inventory for a period of time.

In our analysis, we have assumed that the time at which the reaction period ends is observed. If this assumption does not hold, one may need to introduce sampling to estimate when the reaction period ends. Incorporating sampling into the optimization problem remains a challenging open problem.

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