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Decision Support

A discontinuous mispricing model under asymmetric information

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ABSTRACT

We study a discontinuous mispricing model of a risky asset under asymmetric information where jumps in the asset price and mispricing are modelled by Lévy processes. By contracting the filtration of the informed investor, we obtain optimal portfolios and maximum expected utilities for the informed and uninformed investors. We also discuss their asymptotic properties, which can be estimated using the instantaneous centralized moments of return. We find that optimal and asymptotic utilities are increased due to jumps in mispricing for the uninformed investor but the informed investor still has excess utility, provided there is not too little or too much mispricing.

1. Introduction

Asymmetric information models assume that there are two types of investors in the market: informed and uninformed. The informed investors (e.g., institutional investors with internal research capabilities) observe both fundamental and market prices, while uninformed investors (e.g., retail investors who rely on public information in order to make investment choices) observe market prices only. The uninformed investors are viewed as liquidity traders or hedgers. The prevalence of informed traders affects liquidity, transaction costs, and trading volumes. The informed investors partially reveal information through trades, which can cause higher permanent price changes. Asymmetric information asset pricing models rely on a noisy rational expectation equilibrium in which prices only partially reveal the informed investor's information. Many empirical studies confirm that information asymmetry is priced and imply that liquidity is a primary channel that links information asymmetry to prices (see, e.g., Admati, 1985; Easley & O'Hara, 2004; Grossman & Stiglitz, 1980; Hellwig, 1980; Kelly & Ljungqvist, 2012; Wang, 1993).

Mispricing is the difference between the asset's market price and fundamental value. The fundamental value can be defined as the market price that would prevail if all the market participants were perfectly informed investors. Because of the mean-reverting nature of the mispricing process, it is typically modelled by a continuous Ornstein–Uhlenbeck (O–U) process, while the price of the risky asset is usually modelled by a continuous geometric Brownian motion (see, e.g., Buckley, Brown, & Marshall, 2012; Guasoni, 2006; Wang, 1993). The mean-reverting speed or equivalently, the mean reverting-time, is a proxy for mispricing. Mean-reversion[1] is well-documented in the empirical financial literature and applies to asset returns, stock prices, currencies/exchange rates, interest rates, commodities, indexes, stock index futures, and options.

This paper addresses how asymmetric information, mispricing, and jumps in both the price of a risky asset and its mispricing affect the optimal portfolio strategies and maximum expected utilities of two distinct classes of rational long-horizon investors in an economy where preference is logarithmic. For the purpose of this exposition, we take the risky asset to be stock, but the model can be applied to any asset (e.g. currencies) where prices are mean-reverting. The investors assume that the risky asset has a fundamental or true value (for example, expected discounted future dividends) as well as a market price. When prices move away from its fundamental value, they always revert to it (see, e.g., LeRoy & Porter, 1981; Poterba & Summers, 1988; Shiller, 1981; Summers, 1986). What do we mean by fundamental value? Allen, Morris, and Postlewaite (1993) posit that the fundamental value of an asset is the present value of the stream of the market value of dividends or services generated by this asset. The fundamental value can also be defined as the market price that would prevail if the cost of gathering and processing information is zero for

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[1] The literature related to mean-reversion in asset prices is very large and includes stocks: Poterba and Summers (1988), Depenya and Gil-Alana (2002), Gropp (2004), Mukherji (2011, 2012); currencies: Engel and Hamilton (1990), Cheung and Lai (1994), Sweeney (2006), Serban (2010); indices: Miller, Muthuswamy, and Whaley (1994), Balvers, Wu, and Gilliland (2000), Balvers and Wu (2006), Caporale and Gil-Alana (2008), Kim, Stern, and Stern (2009), Chen and Kim (2011), Spierdijk, Bikker, and van den Hoek (2012), Malin and Bornholt (2013); index futures: Monoyios and Sarno (2002), Miao, Lin, and Chao (2014).

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all investors. Any of the definitions above will suffice. However, we adopt the caveat in Allen et al. (1993) and take fundamental value to mean the value of an asset in normal use, as opposed to some value it may have as a speculative instrument.

Because mispricing is the difference between the asset's market price and its fundamental value, if the fundamental value or asset price jumps, then it follows that the mispricing will jump as well, provided the jump components are not identical and cancel each other. This is particularly evident in the case of independently driven jump processes (see, e.g., Applebaum, 2004). All recent studies (e.g., Buckley et al., 2012; Buckley, Long, & Perera, 2014; Guasoni, 2006) assume that mispricing is a continuous O–U process. However, in this paper, mispricing is no longer purely continuous. Instead, it jumps and is driven by a mean-reverting O–U process which has a continuous component as well as a discontinuous component generated by a pure-jump Lévy process. As in Buckley et al. (2014), the price of the risky asset is still subject to Levy jumps. We solve this model for both investors and present explicit formulas for their optimal portfolios and maximum expected logarithmic utilities under reasonable assumptions.

Portfolio allocation problems have been extensively studied since the seminal work of Markowitz (1952). Merton's (1971) model is the benchmark of optimal asset allocation in the continuous-time framework. There is a rich plethora of portfolio optimization papers[2] in various settings. We refer to Buckley et al. (2012, 2014) for related literature review and discussion on recent progress on this topic.

In this paper, we find that the optimal portfolio of each investor contains excess risky asset that depends on the Lévy measures of both jump processes (asset price and mispricing), the diffusive volatility, and the level of mispricing, as represented by the mean-reversion speed or time. Under quadratic approximation of the portfolios, we show that excess asset holdings are dependent on the first two instantaneous centralized moments of return (see, e.g., Cvitanic et al., 2008). In addition, the maximum expected utility from terminal wealth for each investor is increased by the presence of jumps in the mispricing. This suggests that investors are better off when mispricing jumps than when it changes continuously.

Further, we show that the uninformed investor obtains excess utility from jumps in mispricing. However, as proven in prior studies by Guasoni (2006) and Buckley et al. (2012, 2014), the informed investor still has positive excess utility over the uninformed investor. This implies that the informed investor gets more utility from the diffusive component driving the asset price process. We also show that the asymptotic excess optimal expected utility from terminal wealth of the informed investor has a similar structure to that presented in Guasoni (2006) and Buckley et al. (2012, 2014), but is increased by a factor directly attributed to the jumps in the mispricing. As in the case of jumps in stock price only, the mean-reversion rate λ is replaced by a smaller adjusted mean-reversion rate $\tilde{\lambda}$, which depends on the volatility of the asset price only.

There is also an equivalent representation of excess utility by way of the mean-reversion speed $\hat{\lambda}$ of an adjusted continuous O–U mispricing process which also depends on the volatility of the long-run asset price. In this framework, the excess optimal expected utility of the informed investor is greater (less) than its continuous Merton (1971) geometric Brownian motion counterpart if the product of the diffusive variance and the quadratic variation of the mispricing process is greater (less) than the second instantaneous centralized moment of return of the jump component of the asset price process. Notwithstanding the presence of asymmetric information, mispricing and jumps, our results show that it still pays to be more informed in the long-run, unless there is too little or too much mispricing. Moreover, our results nest those contained in Guasoni (2006) and Buckley et al. (2014).

The practical, financial, economic and operational implication of this paper is that when asymmetric information and jumps exist in both asset price and mispricing, informed and uninformed long-horizon investors will maximize their expected logarithmic utilities from terminal wealth by holding portfolios that contain excess asset holdings which depend not only on the level of information asymmetry and preference, but also on the nature and frequency of the jumps in both asset price and mispricing as dictated by the governing Lévy measures. To the best of our knowledge, this paper is the first to study the utility and portfolio implications for the risky asset by investors when asymmetric information, logarithmic preferences, jumps in asset price and mispricing are intertwined, and therefore contributes to the finance and operational research literature.

Our model is related to heterogeneous agents models (HAM), where investors observe the same information but have different beliefs. However, there are two important distinctions. First, switching is not allowed between investor classes, i.e., an uninformed investor is not allowed to become informed, and vice versa. Second, one investor knows more than the other (see, e.g., Chiarella, Dieci, & He, 2009 and He, 2012 for a review of the HAM literature).

As in Buckley et al. (2014), our model is different from insider trading models[3] which use enlargement of filtrations to obtain the optimal portfolio and utility of the insider trader/informed investor. In contrast to insider trading models, we specify the price dynamics of the informed investor in the larger filtration, and then obtain the dynamics for the uninformed investor by contracting (or restricting) the larger filtration. We then compute and compare optimal portfolios and expected logarithmic utilities for each investor relative to their respective filtrations.

The remainder of the paper is organized as follows. The model is introduced in Section 2, including information flows/filtration and asset price dynamics of investors. In Section 3, expected utilities are maximized, and optimal portfolios are computed and estimated using instantaneous centralized moments of returns. Asymptotic and excess utilities are presented in Section 4. We conclude in Section 5 by giving directions of possible future research. All proofs are given in Appendix A.

2. The discontinuous mispricing model

The economy consists of two assets—a risk-free asset $\mathbf{B}$, called bank account or money market (or U.S. Treasury bill), and a risky asset S, called stock. The risk-free asset earns a continuously compounded risk-free interest rate r_t, and has price $\mathbf{B}_t = \exp\left(\int_0^t r_s\, ds\right)$. The continuous component of the stock's percentage appreciation rate is μ_t, at time $t \in [0, T]$, where $T > 0$ is the investment horizon. The stock is subject to volatility $\sigma_t > 0$. The market parameters are $\mu_t, r_t, \sigma_t, t \in [0, T]$, and are assumed to be **deterministic** functions. The stock's **Sharpe ratio** or **market price of risk** $\theta_t = \frac{\mu_t - r_t}{\sigma_t}$ is square integrable. The risky asset has price S, and lives on a probability space $(\mathbf{\Omega}, \mathcal{F}, \mathbf{P})$ on which is defined two independent standard Brownian motions $W = (W_t)_{t\geq 0}$ and $B = (B_t)_{t\geq 0}$. The stock is viewed by investors in disjoint classes populated by uninformed and informed investors, indexed by $i = 0$ and $i = 1$, respectively. Investors have filtrations $\mathcal{K}_t^i$

[2] See Mansini and Speranza (1999), Kellerer, Mansini, and Speranza (2000), Soyer and Tanyeri (2006), Celikyurt and Ozekici (2007), Corazza and Favaretto (2007), Cvitanic, Polimenis, and Zapatero (2008), Lin and Liu (2008), Canakoglu and Ozekici (2010), Fu, Lari-Lavassani, and Li (2010), Huang, Zhu, Fabozzi, and Fukushima (2010), Yu, Takahashi, Inoue, and Wang (2010), and Buckley et al. (2012, 2014).

[3] See, e.g., Pikovsky and Karatzas (1996), Amendinger, Imkeller, and Schweizer (1998), Amendinger, Becherer, and Schweizer (2003), Ankirchner, Dereich, and Imkeller (2006), Ankirchner and Imkeller (2007), who solve this problem for logarithmic and power utilities. Grorud (2000) studies insider trading in a discontinuous market, while Goll and Kallsen (2003) give a complete explicit characterization of log-optimal portfolios without constraints.

with $\mathcal{K}_t^0 \subset \mathcal{K}_t^1 \subset \mathcal{F}$, $t \in [0, T]$. All random objects are defined on a filtered probability space $(\boldsymbol{\Omega}, \mathcal{F}, \mathcal{K}^i, \mathbf{P})$.

Based on the classical Black–Scholes geometric Brownian motion model (cf. Black & Scholes, 1973) and Merton jump-diffusion model (cf. Merton, 1971, 1976) as well as some recent models discussed in Kou (2002), Guasoni (2006) and Buckley et al. (2014), we propose a new and more general model for the risky asset by incorporating mean-reversion, mispricing, asymmetric information, and Lévy jumps. This new model will be more flexible and help capture the asset price movements including sudden jumps in both the fundamental value (through the mispricing process) and market price. It will also greatly enhance our understanding of the link between asset mispricing and asymmetric information in financial markets. We therefore extend the theory on mispricing models under asymmetric information in Lévy markets to include the situation where both stock price and mispricing jump.

We model the jumps in the mispricing by a pure jump Lévy process $Z = (Z_t)_{t\geq 0}$ with $\mathbf{E}(Z_t) = 0$ and $\mathbf{E}(Z_t^2) < \infty$, for all $t \in [0, T]$. Jumps in the stock price are driven by an independent pure-jump Lévy process $X = (X_t)_{t\geq 0}$. Both jump processes are governed by Poisson random measures N_S and N_U defined on $\mathbf{R}_+ \times \mathcal{B}(\mathbf{R} - \{0\})$ that are linked to the stock (S) and mispricing (U), respectively. These random measures count the jumps of the stock price S (equivalently, X) and mispricing U (equivalently, Z), respectively, in the time interval $(0, t)$. The respective Lévy measures are $\nu_S(dx) = \mathbf{E}N_S(1, dx)$ and $\nu_U(dz) = \mathbf{E}N_U(1, dz)$. We use $\widetilde{N}_U(dt, dz) \triangleq N_U(dt, dz) - \nu_U(dz)dt$ to denote the compensated Poisson measure.

2.1. The model

The risky asset or stock has price S, with log-return dynamics

$$d(\log S_t) = \left(\mu_t - \frac{1}{2}\sigma_t^2\right) dt + \sigma_t\, dY_t + dX_t, \quad t \in [0, T] \tag{1}$$

$$Y_t = p\,W_t + q\,U_t, \quad p^2 + q^2 = 1, p \geq 0,\ q \geq 0, \tag{2}$$

$$dU_t = -\lambda\, U_t\, dt + dL_t, \quad \lambda > 0,\ U_0 = 0, \tag{3}$$

$$L_t = B_t + Z_t, \tag{4}$$

$$X_t = \int_0^t \int_{\mathbf{R}} x\, N_S(dt, dx), \tag{5}$$

$$Z_t = \int_0^t \int_{\mathbf{R}} z\, \widetilde{N}_U(dt, dz), \quad \mathbf{E}(Z_t) = 0,\ \mathbf{E}(Z_t^2) < \infty. \tag{6}$$

W and B are independent standard Brownian motions. $U = (U_t)$ is a mean-reverting **Ornstein–Uhlenbeck** process with mean-reversion rate λ. Because of the exponential decay associated with mean-reverting processes, its half-life is $H = \frac{\ln 2}{\lambda}$. Half-life is a measure of the slowness of a mean-reverting process, and is measured in trading days. It may be defined as the average time it takes the mean-reverting process to get pulled half-way back to its long-term mean. In this context, half-life is a very important number—it gives an estimate of how long we should expect the mispricing to remain far from zero. Thus, a half-life of 10 days means that it takes 20 trading days on average for the mispricing to revert to zero. Noting that half-life is inversely proportion to the mean-reversion rate, we simply use $\delta = \frac{1}{\lambda}$ as the measure of mispricing. Thus, half-life H, λ and δ are equivalent measures of mispricing. Large values of the mean-reversion time δ (equivalently, small values of λ) correspond to high levels of mispricing—that is, the mispricing process U takes a long time to revert to zero, and as such, indicates that mispricing is persistent. The jumps in the mispricing U_t are controlled by the pure-jump Lévy process Z_t, although it also has a continuous component B_t.

Standing assumption. We assume that N_S and N_U are independent random measures if $N_S \neq N_U$. That is, X and Z are independent pure-jump Lévy processes with $\mathbf{E}(Z_t) = 0$, for all $t \in [0, T]$ and $\int_{\mathbf{R}} z\nu_U(dz) < \infty$. From (6),

$$Z_t = \int_0^t \int_{\mathbf{R}} z\widetilde{N}_U(ds, dz) = \int_0^t \int_{\mathbf{R}} zN_U(ds, dz) - t\int_{\mathbf{R}} z\nu_U(dz).$$

Clearly $Z = (Z_t)$ is a martingale provided $\int_{\mathbf{R}} z\nu_U(dz) < \infty$. From Eqs. (3) and (4), we get the dynamics

$$dU_t = -\lambda\, U_t dt + dB_t + dZ_t, \tag{7}$$

which admits the unique solution

$$U_t =\triangleq U_t^B + U_t^Z, \tag{8}$$

where

$$U_t^B = \int_0^t e^{-\lambda(t-s)} dB_s \ \text{ and } \ U_t^Z = \int_0^t e^{-\lambda(t-s)} dZ_s. \tag{9}$$

Note, in this case, that the mispricing process U is a linear combination of two independent O–U processes: a continuous component U^B driven by the Brownian motion B, which is identical to the mispricing process used by Buckley et al. (2014) and Guasoni (2006), and a jump component U^Z driven by the pure-jump martingale Z. It is easy to show that for all $\lambda > 0$ and $t \in [0, T]$, $\mathbf{E}(U_t^B) = 0$ and $\mathbf{E}(U_t^B)^2 = \frac{1-e^{-2\lambda t}}{2\lambda}$. We have a similar result for the process U^Z.

Proposition 1. *Let $\lambda > 0$. For each $t \in [0, T]$, $\mathbf{E}(U_t^Z) = 0$,*

$$\mathbf{Var}(U_t^Z) = \frac{1 - e^{-2\lambda t}}{2\lambda} \int_{\mathbf{R}} z^2\, \nu_U(dz),$$

$$\text{and } \lim_{t\to\infty} \mathbf{Var}(U_t^Z) = \frac{1}{2\lambda} \int_{\mathbf{R}} z^2\, \nu_U(dz).$$

We have a similar result for the mean-reverting mispricing process U.

Proposition 2. *Let $\lambda > 0$. For each $t \in [0, T]$, $\mathbf{E}(U_t) = 0$,*

$$\mathbf{Var}(U_t) = \left(\frac{1 - e^{-2\lambda t}}{2\lambda}\right) \left(1 + \int_{\mathbf{R}} z^2\, \nu_U(dz)\right),$$

$$\text{and } \lim_{t\to\infty} \mathbf{Var}(U_t) = \frac{1}{2\lambda} \left(1 + \int_{\mathbf{R}} z^2\, \nu_U(dz)\right).$$

It now follows from Eqs. (2) and (8) that

$$Y_t = p\,W_t + q\,U_t = p\,W_t + q\int_0^t e^{-\lambda(t-s)} dB_s + q\int_0^t e^{-\lambda(t-s)} dZ_s,$$

is a stochastic process consisting of three independent components: two are continuous, while one is a pure-jump Lévy process. Taking differentials and importing Eq. (7) yield

$$\begin{aligned} dY_t &= p\,dW_t + q\,dU_t = p\,dW_t + q(-\lambda U_t dt + dB_t + dZ_t) \\ &= p\,dW_t + q\,dB_t - \lambda q U_t dt + q\,dZ_t \\ &= p\,dW_t + q\,dB_t - \lambda q U_t^B dt - \lambda U_t^Z dt + q\,dZ_t \\ &= dB_t^1 + \upsilon_t^{1,B} dt + \upsilon_t^Z dt + q\,dZ_t = dB_t^1 + \upsilon_t^1 dt + q\,dZ_t, \end{aligned} \tag{10}$$

where

$$B_t^1 \triangleq p\,W_t + q\,B_t, \tag{11}$$

$$\upsilon_t^1 \triangleq \upsilon_t^{1,B} + \upsilon_t^Z, \quad \upsilon_t^{1,B} \triangleq -\lambda\, q\, U_t^B, \quad \upsilon_t^Z \triangleq -\lambda\, q\, U_t^Z. \tag{12}$$

2.2. Filtration of the investors

We expand the filtration developed in Buckley et al. (2012, 2014) and Guasoni (2006) to include the information generated by $X = (X_t)$ and the martingale $Z = (Z_t)$. We assume that all filtrations obey the usual hypothesis. Namely, they are right-continuous and complete. Let $\mathcal{F}_t^i$ $(i = 1, 0)$ be the information flow/filtration for the informed and uninformed investors in the models where there are no jumps in mispricing or asset price (see Buckley et al., 2012 or Guasoni, 2006). These filtrations are defined as

$$\mathcal{F}_t^1 \triangleq \sigma\,(W_s, B_s : s \le t) = \mathcal{F}_t^{S,U} \quad \text{and} \quad \mathcal{F}_t^0 \triangleq \sigma\,(Y_s : s \le t) = \mathcal{F}_t^Y.$$

The filtration of the informed investor in our new framework is $\mathcal{K}^1 = (\mathcal{K}_t^1)_{t\ge 0}$ defined by

$$\mathcal{K}_t^1 = \mathcal{F}_t^1 \vee \sigma\,(Z_u, X_u : u \le t), \quad t \in [0,\, T].$$

The uninformed investor observes the stock price only and does not know what the mispricing is, if it exists. Uninformed investors have filtration $\mathcal{K}^0$, where $\mathcal{K}_t^0 \subset \mathcal{F}$ is defined by the prescription:

$$\mathcal{K}_t^0 = \mathcal{F}_t^0 \vee \sigma\,(Z_u, X_u : u \le t), \quad t \in [0,\, T],$$

which is contained in $\mathcal{K}_t^1$, the filtration of the informed investor.

We shall develop the dynamics for these investors, starting with the log-return dynamics for the stock given by Eqs. (1)–(6). Recall that $U_t = U_t^B + U_t^Z$.

Definition 1. Define the continuous part of the process Y by

$$Y_t^c \triangleq Y_t - q\,U_t^Z = p\,W_t + q\,U_t^B. \tag{13}$$

We have the following important result.

Lemma 1. *There exist an $\mathcal{K}^0$-Brownian motion B^0, and a process $\upsilon_t^{0,B} \equiv -\phi_t^{0,B}$, adapted to $\mathcal{K}_t^0$, such that for each $t \in [0, T]$ and $q \in [0,\, 1]$,*

$$B_t^0 = Y_t^c + \int_0^t \phi_s^{0,B} ds \equiv Y_t - q\,U_t^Z - \int_0^t \upsilon_s^{0,B} ds, \tag{14}$$

where $\lambda > 0$ is the mean-reversion speed of U, $p^2 + q^2 = 1$ and

$$\upsilon_t^{0,B} = -\lambda \int_0^t e^{-\lambda(t-s)}(1+\gamma(s))dB_s^0, \tag{15}$$

$$\gamma(s) = \frac{1-p^2}{1+p\tanh(\lambda p s)} - 1,\ p \in [0,\, 1].$$

From Eq. (14), we have $Y_t = B_t^0 + \int_0^t \upsilon_s^{0,B}\,ds + q\,U_t^Z$. Therefore,

$$\begin{aligned} dY_t &= dB_t^0 + \upsilon_t^{0,B}\,dt + q\,dU_t^Z = dB_t^0 + \upsilon_t^{0,B}\,dt - \lambda\,q\,U_t^Z\,dt + qdZ_t \\ &= dB_t^0 + \upsilon_t^{0,B}\,dt + \upsilon_t^Z\,dt + qdZ_t = dB_t^0 + \upsilon_t^0\,dt + qdZ_t, \end{aligned} \tag{16}$$

where

$$\upsilon_t^0 \triangleq \upsilon_t^{0,B} + \upsilon_t^Z, \quad \upsilon_t^{0,B} \triangleq -\lambda \int_0^t e^{-\lambda(t-s)}(1+\gamma(s))dB_s^0,$$

$$\upsilon_t^Z \triangleq -\lambda\,q\,U_t^Z. \tag{17}$$

Note that (16) has the same form as (10). We now extract Proposition 2 in Buckley et al. (2012) concerning $\upsilon_t^{i,B}$, the number of standard deviations from the mean return μ_t, since it will be required in the sequel.

Proposition 3. *Let $\lambda > 0$, $i \in \{0, 1\}$, $p \in [0,\, 1]$, $p^2 + q^2 = 1$ and $t \in [0,\, T]$.*

Let $\upsilon_t^{0,B} = -\lambda \int_0^t e^{-\lambda(t-s)}(1+\gamma(s))dB_s^0$ and $\upsilon_t^{1,B} = -\lambda\,q\,U_t$. Then

(0) $\mathbf{E}[\upsilon_t^{i,B}] = 0$.

(1) $\mathbf{Var}\upsilon_t^{0,B} = \mathbf{E}[\upsilon_t^{0,B}]^2 = \lambda^2 \int_0^t e^{-2\lambda(t-s)}(1+\gamma(s))^2 ds \le \frac{1}{2}\lambda\,q^2$.

(2) $\mathbf{Var}\upsilon_t^{1,B} = \mathbf{E}[\upsilon_t^{1,B}]^2 = \frac{\lambda}{2}\,(1-p^2)\,(1-e^{-2\lambda t}) \le \frac{1}{2}\lambda\,q^2$.

(3) $\mathbf{Var}\upsilon_t^{i,B} = \mathbf{E}[\upsilon_t^{i,B}]^2 \to \frac{\lambda}{2}(1-p)(1+(-1)^{i+1}p):$ *as* $t \to \infty$.

(4) $\int_0^T \mathbf{E}[\upsilon_t^{i,B}]^2 dt \simeq \frac{\lambda}{2}\,(1-p)(1+(-1)^{i+1}p)T$ *as* $T \longrightarrow \infty$.

(5) *As $T \longrightarrow \infty$, the asymptotic excess cumulative variance of the υ^is is*

$$\int_0^T \mathbf{E}[\upsilon_t^{1,B}]^2 dt - \int_0^T \mathbf{E}[\upsilon_t^{0,B}]^2 dt \simeq \lambda\,p\,(1-p)\,T \le \frac{\lambda}{4}\,T.$$

We have a useful result for the process υ^i, which is a generalization of the process used in Buckley et al. (2012, 2014) and Guasoni (2006).

Proposition 4. *Let $\lambda > 0, t \in [0,\, T],\ i \in \{0, 1\}$ and $\upsilon_t^i = \upsilon_t^{i,B} + \upsilon_t^Z$. Then for $p,\ q \in [0, 1]$ with $p^2 + q^2 = 1$, we have*

(1) $\mathbf{E}[\upsilon_t^i] = 0$.

(2) $\mathbf{Var}[\upsilon_t^1] = \frac{\lambda}{2}(1 - e^{-2\lambda\,t})(1+p)(1+(-1)^1 p)\,(1 + \int_{\mathbf{R}} z^2\,\nu_U(dz))$.

(3) $\mathbf{Var}[\upsilon_t^0] = \lambda^2 \int_0^t e^{-2\lambda(t-s)}(1+\gamma(s))^2 ds +$
$\lambda^2\,q^2\,\left(\frac{1-e^{-2\lambda\,t}}{2\lambda}\right)\int_{\mathbf{R}} z^2\,\nu_U(dz)$.

Moreover,

$$\lim_{t\to\infty} \mathbf{E}[\upsilon_t^i]^2 = \frac{\lambda}{2}(1+p)(1+(-1)^i p)\left(1 + \int_{\mathbf{R}} z^2\,\nu_U(dz)\right).$$

2.3. Asset price dynamics for the investors

Let $i \in \{0, 1\}$. Define $\mu_t^* \triangleq \mu_t - \frac{1}{2}\sigma_t^2$. The dynamics for the stock is given by (1) as $d(\log S_t) = \mu_t^*\,dt + \sigma_t\,dY_t + dX_t$, $t \in [0,\, T]$. Imposing (2), (3) and (10) yields

$$\begin{aligned} d(\log S_t) &= \mu_t^* dt + \sigma_t(dB_t^i + \upsilon_t^i dt + q\,dZ_t) + dX_t \\ &= \mu_t^* dt + \upsilon_t^i \sigma_t dt + \sigma_t dB_t^i + q\,\sigma_t dZ_t + dX_t \\ &= \left(\mu_t^* - q\sigma_t \int_{\mathbf{R}} x\upsilon_U(dx) + \upsilon_t^i \sigma_t\right) dt + \sigma_t dB_t^i \\ &\quad + \int_{\mathbf{R}} q\,\sigma_t z N_U(dt, dz) + \int_{\mathbf{R}} x N_S(dt, dx) \\ &= \mu_t^{*,i} dt + \sigma_t dB_t^i + \int_{\mathbf{R}} q\,\sigma_t z N_U(dt, dz) + \int_{\mathbf{R}} x N_S(dt, dx), \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mu_t^{*,i} &= \mu_t^* - q\sigma_t \int_{\mathbf{R}} z\upsilon_U(dz) + \upsilon_t^i \sigma_t \\ &= \mu_t + \upsilon_t^i \sigma_t - q\sigma_t \int_{\mathbf{R}} z\upsilon_U(dz) - \frac{1}{2}\sigma_t^2. \end{aligned} \tag{19}$$

Remark 1. If $N_S = N_U = N$ then $\nu = \nu_U = \nu_S$, and (18) becomes

$$d(\log S_t) = \mu_t^{*,i} dt + \sigma_t\,dB_t^i + \int_{\mathbf{R}} x\,(q\sigma_t + 1)N(dt, dx). \tag{20}$$

We now quote a very useful result on page 47 in Applebaum (2004).

Lemma 2 (Proposition 1.3.12, Applebaum (2004)). *If $(N_1(t), t \ge 0)$ and $(N_2(t), t \ge 0)$ are two independent Poisson processes defined on the same probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with arrival times $(T_n^{(j)},\ n \in \mathbf{N})$, for $j = 1, 2$, respectively, then $\mathbf{P}(T_m^{(1)} = T_n^{(2)}$ for some $m, n \in\ \mathbf{N}) = 0$.*

This result tells us that **independent Poisson processes do not jump at the same time**. Thus, for random measures N_U and N_S and Borel set $A \in \mathcal{B}(\mathbf{R} - \{0\})$, $N_U(t, A)$ and $N_S(t, A)$ are independent Poisson processes and do not jump together by Lemma 2. We have the following important result.

Theorem 1. *Let $i \in \{0,\, 1\}$. The log-return dynamics (1), for the ith investor, is*

$$d(\log S_t) = \mu_t^{*,i} dt + \sigma_t dB_t^i + \int_{\mathbf{R}} q\sigma_t\,z\,N_U(dt, dz) + \int_{\mathbf{R}} x\,N_S(dt, dx), \tag{21}$$

and its percentage return dynamics is

$$\begin{aligned} \frac{dS_t}{S_{t-}} &= \mu_t^i dt + \sigma_t dB_t^i + \int_{\mathbf{R}} (e^{q\sigma_t z} - 1)N_U(dt, dz) \\ &\quad + \int_{\mathbf{R}} (e^x - 1)N_S(dt, dx), \end{aligned} \tag{22}$$

where

$$\mu_t^{*,i} \triangleq \mu_t^i - \frac{1}{2}\sigma_t^2, \quad \mu_t^i \triangleq \mu_t + \upsilon_t^i \sigma_t - q\sigma_t \int_{\mathbf{R}} z\upsilon_U(dz).$$

The asset's percentage return dynamics (22) consists of four components: the first is continuous; the second is random and driven by a standard Brownian motion; the third is driven by jumps in the mispricing process and the fourth component is driven by jumps in the asset price. However the total mean return is driven only by three components: the first is continuous, the second is due to the jumps in mispricing, and the third is due to jumps in the asset price.

3. Optimal portfolios and expected utilities

Random objects for the ith investor live on a filtered space $(\mathbf{\Omega}, \mathcal{F}, \mathcal{K}^i, \mathbf{P})$ for $i \in \{0, 1\}$.

Definition 2 (Portfolio process)**.** The process $\pi \equiv \pi^i : [0, T] \longrightarrow \mathbf{R}$ is called the portfolio process of the ith investor, if $\pi^i_t(\omega)$ is $\mathcal{K}^i_t$-adapted for each $\omega \in \Omega$ and $\mathbf{E} \int_0^T (\pi^i_t \sigma_t)^2 dt < \infty$.

Note that π^i_t is really $\pi^i_t(\omega)$, where $\omega \in \Omega$, and hence, is a random process. π^i_t is the proportion of the wealth of the ith investor that is invested in the risky asset at time $t \in [0, T]$. The remainder, $1 - \pi^i_t$, is invested in the risk-free asset or money market. Where it is clear, we drop the superscript "i" and simply use π for the portfolio process.

Definition 3 (The wealth processes)**.** The wealth process for the ith investor is $V^{i,\pi,x} : [0, T] \longrightarrow \mathbf{R}$, where $V^{i,\pi,x}_t$ is the value of the portfolio consisting of the risky asset and money market at time t, when π^i_t is invested in the risky asset and the initial capital is $x > 0$.

For brevity, we denote this process by $V^{i,\pi} = (V^{i,\pi}_t)$ or simply $V^i = (V^i_t)$, $t \in [0, T]$, when the context is clear. The initial capital is $V^i_0 = x > 0$.

Definition 4 (Admissible portfolio)**.** π is called an admissible portfolio if $V^\pi_t > 0$ almost surely, for all $t \in [0, T]$.

3.1. Maximization of logarithmic utility from terminal wealth

Let $\pi \equiv \pi^i \equiv (\pi^i_t)$, $t \in [0, T]$, be the portfolio process of the ith investor. Let $\theta^i = \frac{\mu^i - r}{\sigma}$ be the Sharpe ratio. The corresponding wealth process with initial capital $x > 0$ is: $V^i_t \equiv V^{i,\pi}_t \equiv V^{i,\pi,x}_t$. We now give the dynamics for V^i.

Theorem 2. *The percentage return of the wealth V^i of the ith investor with stock price dynamics (1) is:*

$$\begin{aligned}\frac{dV^i_t}{V^i_{t-}} &= (r_t + \pi_t \sigma_t \theta^i_t)dt + \pi_t \sigma_t dB^i_t + \int_{\mathbf{R}} \pi_t (e^{q\sigma_t z} - 1) N_U(dt, dz) \\ &\quad + \int_{\mathbf{R}} \pi_t (e^x - 1) N_S(dt, dx).\end{aligned} \tag{23}$$

The discounted wealth process is

$$\begin{aligned}\widetilde{V}^i_t &= V^i_t \exp\left(-\int_0^t r_s ds\right) \\ &= V^i_0 \exp\left(\int_0^t \left(\pi_s \sigma_s \theta^i_s - \frac{1}{2}\pi_s^2 \sigma_s^2\right) ds + \int_0^t \pi_s \sigma_s dB^i_s\right) \\ &\quad \times \Pi_{0 \le u \le t}(1 + \pi_u (e^{q\sigma_u \Delta Z_u} - 1)) \times \Pi_{0 \le s \le t}(1 + \pi_s (e^{\Delta X_s} - 1)).\end{aligned} \tag{24}$$

Remark 2. We use the subscripts "u" and "s" in Eq. (24) to distinguish the jumps due to mispricing U and asset price S, respectively. The budget constraint (23) consists of four components: the first is continuous; the second is random and drive by a standard Brownian motion; the third is driven by jumps in the mispricing process and the fourth component is driven by jumps in the asset price. Note that volatility plays a key role in the component driven by the mispricing process, while the component driven by the jumps in asset price is independent of volatility. The discounted wealth process has identical dependencies. This will be the argument used to maximize the expected logarithmic utility function.

3.2. Admissible set for each investor

Let $V^i_0 = x$. We seek a portfolio process $\pi^i = (\pi_t)^i_{t \ge 0}$ in an admissible set $\mathcal{A}_i(x)$, defined by

$$\mathcal{A}_i(x) = \{\pi^i : \pi^i \text{ is } \mathcal{K}^i - \text{predictable such that } \widetilde{V}^{i,\pi^i}_t > 0 \text{ a.s. } \forall t \in [0, T]\}.$$

π^i is predictable if it is measurable with respect to the predictable sigma–algebra on $[0, T] \times \Omega$, which is the sigma–algebra of all left continuous functions with right limits on $[0, T] \times \Omega$. The optimal portfolio for the informed or uninformed investor is $\pi^{*,i} \in \mathcal{A}_i(x)$ such that

$$\mathbf{E}(\log \widetilde{V}^{i,\pi^{*,i}}_T) = \max_{\pi \in \mathcal{A}_i(x)} \mathbf{E} \log \widetilde{V}^{i,\pi}_T.$$

That is, $\pi^{*,i} = \arg\max_{\pi \in \mathcal{A}_i(x)} \mathbf{E} \log \widetilde{V}^{i,\pi}_T$. Since $\theta_t \equiv \theta^i_t$ is independent of π_t, observe that $\mathbf{E} \log \widetilde{V}^{i,\pi}_T$ is maximized iff $\mathbf{E} \int_0^T f^{(i)}_{U,S}(\pi^i_t) dt$ is maximized. That is, if $f^{(i)}_{U,S}(\pi^i)$ defined by (31) below is maximized on the admissible set $\mathcal{A}_i(x)$. This approach is similar to the optimization method used by Amendinger et al. (1998), Ankirchner et al. (2006), and Liu, Longstaff, and Pan (2003).

3.3. The objective function G

In order to obtain optimal portfolio strategies, we need the assistance of an objective function G. For any Lévy measure $\nu_a(\cdot)$, $a \in \{U, S\}$, define the (partial) objective function G_a by the prescription $G_a : [0, 1] \longrightarrow \mathbf{R}$,

$$\begin{aligned}G_a(\alpha\,;\, s) &\triangleq \int_{\mathbf{R}} \log(1 + \alpha(e^{sx} - 1)) \nu_a(dx), \ \alpha \in [0, 1], \\ s &\in I_\sigma \triangleq (0, \sigma_{\max}] \cup \{1\},\end{aligned} \tag{25}$$

where s is a non-negative parameter or function representing volatility and $\sigma_{\max}$ is positive. Note that α, and hence π, is restricted to the domain $[0,1]$.

Standing assumption. We assume that an integer $k \ge 2$ exists such that

$$\int_{\mathbf{R}} (e^{\pm sx} - 1)^k \nu_a(dx) < \infty, \ \text{where } a \in \{U, S\}, \ s \in I_\sigma.$$

Define the objective function $G : [0, 1] \longrightarrow \mathbf{R}$ by

$$G(\alpha\,;\, s) \triangleq G_U(\alpha\,;\, s) + G_S(\alpha\,;\, 1). \tag{26}$$

Lemma 3. *Let $s \in I_\sigma$. If $\alpha \in [0, 1]$, then $G''(\alpha\,; s) < 0$.*

This result proves that G is strictly concave on $[0, 1]$ for each fixed volatility level s, and hence, will admit unique optimal portfolios. We now give a major result for optimal portfolios strategies of investors.

Theorem 3. *Let $i \in \{0, 1\}$ and $q \in [0, 1]$. For the ith investor, the optimal portfolio π^i, that maximizes the expected logarithmic utility from terminal wealth over the investment period $[0, T]$, is given by*

$$\pi^i_t = \frac{\mu^i_t - r_t + G'(\pi^i_t\,;\, q\sigma_t)}{\sigma_t^2} \equiv \frac{\mu^i_t - r_t + G'_U(\pi^i_t\,;\, q\sigma_t) + G'_S(\pi^i_t\,;\, 1)}{\sigma_t^2}. \tag{27}$$

In terms of the stock's total return b^i_t, the optimal portfolio is

$$\pi^i_t = \frac{b^i_t - r_t - K_U(q\sigma_t) - K_S(1) + G'(\pi^i_t; q\sigma_t)}{\sigma_t^2}, \tag{28}$$

where

$$K_a(s) = \int_{\mathbf{R}} (e^{sx} - 1)\nu_a(dx), \quad a \in \{U, S\}, \quad s \ge 0. \tag{29}$$

The maximum expected logarithmic utility from terminal wealth, with $x > 0$ in initial capital, is

$$u^i(x) = \log x + \frac{1}{2}\mathbf{E}\int_0^T (\theta_t^i)^2 dt + \mathbf{E}\int_0^T f_{U,S}^{(i)}(\pi_t^i)dt, \tag{30}$$

where

$$f_{U,S}^{(i)}(\pi^i) = G(\pi^i; q\sigma) - \frac{1}{2}(\pi^i\sigma - \theta^i)^2, \tag{31}$$

$$G(\pi^i; q\sigma) = G_U(\pi^i; q\sigma) + G_S(\pi^i; 1),$$

$$\mu_t^i = \mu_t + \upsilon_t^i\sigma_t - q\sigma_t\int_{\mathbf{R}} x\upsilon_U(dx),$$

$$\upsilon_t^i = \upsilon_t^{i,B} + \upsilon_t^Z,$$

$$\upsilon_t^{1,B} = -\lambda\, q\, U_t^B, \quad \upsilon_t^{0,B} = -\lambda\int_0^t e^{-\lambda(t-s)}(1+\gamma(s))dB_s^0,$$

$$\upsilon_t^Z = -\lambda\, q\, U_t^Z,$$

$$b_t^i = \mu_t^i + K_U(q\sigma_t) + K_S(1). \tag{32}$$

Remark 3.

(a) The optimal portfolio strategy for each investor consists of three components. First, there is a continuous component which is the Merton (1971) optimal for that investor. The second component is due to jumps in the mispricing process, level of mispricing q and volatility. The third component is due to jumps in the asset price only. One may also present the optimal portfolio strategy of each investor through it total expected return b_t, the derivative of objective function G and kernel, K_a.

(b) Let $u_{T,c}^i(x) = \log x + \frac{1}{2}\mathbf{E}\int_0^T(\theta_t^i)^2 dt$ and $u_{T,d}^i(x) = \mathbf{E}\int_0^T f_{U,S}^{(i)}(\pi_t^i)dt$. Then $u^i(x) \equiv u_T^i(x) = u_{T,c}^i(x) + u_{T,d}^i(x)$, where $u_{T,c}^i(x)$ is the continuous component representing the maximum expected logarithmic utility from terminal wealth for the purely continuous Merton case, and $u_{T,d}^i(x)$ is the discontinuous component which is due to jumps in the mispricing and the asset prices.

Remark 4. Because of the intractability of the integrals $\int_{\mathbf{R}}\log(1 + \pi(e^{\sigma x}-1))v_a(dx)$, where $a \in \{U,S\}$, we compute the optimal portfolio π^i, using approximation techniques. This is done via the instantaneous centralized moments of return for *both* Lévy processes X and Z, with Lévy measures v_S and v_U, respectively.

3.4. The case $N_U = N_S = N$

We now study the case where X and Z, given by (5) and (6), respectively, have the same Poisson random measure N, and hence the same Lévy measure $v = v_U = v_S$. As already demonstrated by (20), the stock has log-return dynamics

$$d(\log S_t) = \mu_t^{*,i}\, dt + \sigma_t\, dB_t^i + \int_{\mathbf{R}} x\,(q\sigma_t + 1)N(dt,dx).$$

Application of Itô's formula yields

$$\frac{dS_t}{S_{t-}} = \mu_t^i\, dt + \sigma_t\, dB_t^i + \int_{\mathbf{R}}(e^{(q\sigma_t+1)x} - 1)N(dt,dx).$$

It follows from Theorem 1, that the dynamics of the wealth process V^i is

$$\frac{dV_t^i}{V_{t-}^i} = (r_t + \pi_t\,\sigma_t\,\theta_t^i)\, dt + \pi_t\,\sigma_t\, dB_t^i + \int_{\mathbf{R}}\pi_t(e^{(q\sigma_t+1)x} - 1)N(dt,dx),$$

with explicit solution

$$V_t^i = V_0^i \exp\left(\int_0^t r_s\, ds + \int_0^t\left(\pi_s\sigma_s\theta_s^i - \frac{1}{2}\pi_s^2\sigma_s^2\right)ds + \int_0^t \pi_s\sigma_s dB_s^i\right) \times \Pi_{0\le s\le t}(1 + \pi_s(e^{(q\sigma_s+1)\Delta X_s} - 1)).$$

The optimal portfolio π^i is given by the equation

$$\pi^i = \frac{\sigma\theta^i + G'(\pi^i; q\sigma)}{\sigma^2} = \frac{\mu^i - r + G'(\pi^i; q\sigma)}{\sigma^2},$$

where

$$G(\alpha; \sigma) = \int_{\mathbf{R}}\log(1 + \alpha(e^{(\sigma+1)x} - 1))v(dx).$$

Observe that the optimal portfolio strategy for each investor now depends on two components: a Merton (1971) type optimal component and a jump component that depends on the jump intensity, level of mispricing, and the volatility of the diffusive component of the price dynamic. The budget constraint also has a similar dependence, in addition to having a diffusive component.

3.5. Approximation of optimal portfolios using instantaneous centralized moments

We now develop the tools required to estimate optimal portfolio strategies of investors. We adopt the definition of instantaneous centralized moments of return as presented in Cvitanic et al. (2008). For the risky asset with price S and positive integer k, we define the kth instantaneous centralized moment by the prescription

$$M_S(k) \triangleq \int_{\mathbf{R}}(e^x - 1)^k v_S(dx) = \sum_{j=1}^k (-1)^{k-j}\binom{k}{j}K_S(j),$$

where the kernel K_S is defined by

$$K_S(s) \triangleq \int_{\mathbf{R}}(e^{sx} - 1)v_S(dx), \quad s \ge 0.$$

For the jump component of the mispricing process U, the instantaneous centralized moments of return are dependent on the volatility σ_t, and are given by:

$$M_U(k; \sigma) \triangleq \int_{\mathbf{R}}(e^{\sigma z} - 1)^k v_U(dz) = \sum_{j=1}^k (-1)^{k-j}\binom{k}{j}K_U(j; \sigma),$$

where the kernels are

$$K_U(j; \sigma) \triangleq \int_{\mathbf{R}}(e^{j\sigma z} - 1)v_U(dz).$$

We use approximation methods to compute the optimal portfolios since the partial objective functions $G_a(\alpha; s) \triangleq \int_{\mathbf{R}}\log(1 + \alpha(e^{sx} - 1)\, v_a(dx)$, $a \in \{U,S\}$, $s \in I_\sigma \equiv (0, \sigma_{\max}] \cup \{1\}$, is in general not tractable. We assume, as before, that $\int_{\mathbf{R}}(e^{\pm sx} - 1)^2 v_a(dx) < \infty$ to ensure that the optimal portfolios exist (i.e.; $G''(\alpha) < 0$). Since $G(\alpha; s) = G_U(\alpha; s) + G_S(\alpha; 1)$, where $\alpha \in [0, 1]$ and $s \in I_\sigma$, then if $\int_{\mathbf{R}}(e^{\pm sx} - 1)^k v_a(dx) < \infty$, we can approximate $G(\alpha; s)$ by a kth degree polynomial $G_k(\alpha; s)$, where

$$G_k(\alpha; s) \triangleq \sum_{j=1}^k (-1)^{j-1}M(j; s)\frac{\alpha^j}{j!}, \tag{33}$$

$$M(j; s) \triangleq M_U(j; s) + M_S(j; 1), \quad j = 1, 2, \cdots, k.,$$

$$M_a(j; s) = \int_{\mathbf{R}}(e^{sx} - 1)^j v_a(dx) = \sum_{r=1}^j (-1)^{r-1}\binom{j}{r}K_a(r; s),$$

$$K_a(j; s) = \int_{\mathbf{R}}(e^{jsx} - 1)\, v_a(dx), \quad a \in \{U,S\}.$$

We have the following lemma.

Lemma 4. *Let $k \in \mathbf{N}$. If $\int_{\mathbf{R}}(e^{\pm sx} - 1)^k v_a(dx) < \infty$, where $s \in I_\sigma$, and $a \in \{U,S\}$, then the kth instantaneous centralized moment exists and is* *given by*

$$M(k\,;\,s)=\sum_{j=1}^{k}(-1)^{j-1}\binom{k}{j}K(j\,;\,s)=M_U(k\,;\,s)+M_S(k\,;\,1),$$

where

$$K(j\,;\,s)=K_U(js)+K_S(j),\ \ K_a(\gamma)=\int_{\mathbf{R}}(e^{\gamma x}-1)\,\nu_a(dx),\quad a\in\{U,\,S\}.$$

Note that the kernel K_a, where $a \in \{U,\ S\}$, is used to calculate the instantaneous centralized moments. Because of the intractable nature of the integrals involved in computing the optimal portfolio strategies, we resort to approximations of these portfolios using the instantaneous centralized moments. The following approximation result is a direct consequence of Theorem 3.

Theorem 4. *Let $i \in \{0,\,1\}$ and π^i be the optimal portfolio that maximizes the expected logarithmic utility from terminal wealth over the investment period $[0,\ T]$. Then $\pi(k)^i$, the approximation of the optimal portfolio based on a kth degree polynomial approximation (33) of the objective function G, is given by*

$$\pi_t(k)^i=\frac{\mu_t^i-r_t+G_k'(\pi_t(k)^i\,;\,q\sigma_t)}{\sigma_t^2}. \tag{34}$$

The maximum expected logarithmic utility from terminal wealth, $u^i(x)$, with $x>0$ as initial capital, is approximated by

$$u(k)^i(x)=\log x+\frac{1}{2}\mathbf{E}\int_0^T(\theta_t^i)^2dt+\mathbf{E}\int_0^T f_{U,S}^{(i)}(\pi_t(k)^i)dt, \tag{35}$$

where $f_{U,S}^{(i)}(\pi^i)$ is defined by (31).

Remark 5. Note from Eq. (34) that the excess stock holding over $\frac{\mu_t^i-r_t}{\sigma_t^2}$, the Merton (1971) optimal, is $\frac{G_k'(\pi_t(k)^i\,;\,q\sigma_t)}{\sigma_t^2}$, which is dependent on $M(j\,;\,s)=M_U(j\,;\,s)+M_S(j\,;\,1), j=1,2,\cdots,k$, the instantaneous centralized moments of return contained in Eq. (33). Each moment is the sum of the instantaneous centralized moments for the mispricing and asset price, respectively.

4. Asymptotic utilities

In this section we discuss the asymptotic results of the optimal expected utilities for both informed and uninformed investors. We set the interest rate $r=0$ without loss of generality.

As before, we assume that the objective function $G(\alpha)\equiv G(\alpha\,;\,s)=G_U(\alpha\,;\,s)+G_S(\alpha\,;\,1)$ is restricted to the domain [0, 1], where $s\in I_\sigma$, and that $\int_{\mathbf{R}}(e^{\pm sx}-1)^2\nu_a(dx)<\infty$, for each $a\in\{U,S\}$. That is, we have at least a quadratic approximation of $G(\alpha)$. Let $M_a(j\,;s)=\int_{\mathbf{R}}(e^{sx}-1)^j\nu_a(dx),\ a\in\{U,\ S\}$. Define

$$M(j\,;\sigma_t)\triangleq M_U(j\,;\sigma_t)+M_S(j\,;1),\quad j=1,2,\cdots,k, \tag{36}$$

$$A_t\triangleq\frac{-M(2\,;q\sigma_t)}{2(\sigma_t^2+M(2\,;\,q\sigma_t))},$$

$$B_t\triangleq\frac{2M(1\,;q\sigma_t)}{2(\sigma_t^2+M(2\,;\,q\sigma_t))},$$

$$C_t\triangleq\frac{M^2(1\,;q\sigma_t)}{2(\sigma_t^2+M(2\,;\,q\sigma_t))}.$$

Note that A, B, and C are functions of the diffusive volatility σ_t. Let $Q(\theta)$ be defined by

$$Q(\theta)\triangleq A\theta^2+B\theta+C, \tag{37}$$

where $\theta=\frac{\mu}{\sigma}$ represents Sharpe ratio. Let

$$\gamma_t^2=\frac{\sigma_t^2}{\sigma_t^2+M(2\,;\,q\sigma_t)}. \tag{38}$$

Then γ_t is the proportion of total volatility due to the diffusive component of the risky asset with dynamics (1). We assume that $\lim_{t\to\infty}\sigma_t=\sigma_\infty=\sigma>0$, and set

$$\gamma^2=\lim_{t\to\infty}\gamma_t^2=\frac{\sigma_\infty^2}{\sigma_\infty^2+M(2\,;\,q\sigma_\infty)}. \tag{39}$$

Theorem 5. *Assume that $\int_{\mathbf{R}}(e^{\pm sx}-1)^k\nu(dx)<\infty, s\in I_\sigma,\ k\geq 2$ and G is restricted to $[0,\ 1]$. Let $x>0$ be the initial wealth of investors and $i\in\{0,1\}$.*

(a) As $T\to\infty$, the asymptotic optimal expected utility for the ith investor due to jumps is

$$u_{T,\,d}^i(x)\sim\int_0^T Q\left(\frac{\mu_t}{\sigma_t}:\sigma_t, M(1;q\sigma_t)M(2;q\sigma_t)\right)dt + \frac{\lambda}{2}A_\infty(1-p)(1+(-1)^{i+1}p)\left[1+\int_{\mathbf{R}}z^2\,\nu_U(dz)\right]T, \tag{40}$$

where

$$A_\infty=\lim_{t\to\infty}A_t=-\frac{M(2;q\sigma_\infty)}{2(\sigma_\infty^2+M(2;q\sigma_\infty))}. \tag{41}$$

(b) The asymptotic excess optimal expected utility of the uninformed investor relative to the informed due to jumps is strictly positive and given by

$$u_{T,\,d}^0(x)-u_{T,\,d}^1(x)\sim-\lambda\,A_\infty\,p\,(1-p)\left[1+\int_{\mathbf{R}}z^2\,\nu_U(dz)\right]T. \tag{42}$$

Observe that the optimal expected utility is the sum of two disjoint components—one is continuous, while the other is discrete. $u_{T,\,d}^i(x)$ is the discrete component of optimal expected utility. It is driven by the jump processes only. Part (a) shows that the jump component of optimal expected utility depends on the diffusive volatility σ_t, first and second instantaneous centralized moments, $M(1;q\sigma_t)$ and $M(2;q\sigma_t)$, the level of mispricing q, the jump intensity of the mispricing process, and the long-run proportion of total volatility due to the diffusive component of the asset price dynamics. Part (b) gives the excess optimal expected utility of the uninformed investor due to jumps only. It depends on the proportion of mispricing, the mean-reversion speed, the variance of the mispricing process, and a long-run variance ratio. This excess is positive because A_∞, the long-run proportion of diffusive variance to total variance of the percentage return of the risky asset is **always negative**, unless there is no diffusive component, in which case it is zero. This is an unexpected result, which suggests that jumps favour the uninformed investor from a utility standpoint. However, as the next major result shows, the informed investor still has positive excess utility relative to the uninformed investor. This implies that the informed investor gains most of his/her utility from the diffusive component of the asset price dynamics, and that it pays to be more informed unless there is too little or too much mispricing. We have the following major result as a consequence of the quadratic approximation of $G(\alpha\,;\,\sigma)$.

Theorem 6. *Assume that the conditions of Theorem 5 hold. Let the investment horizon $T\longrightarrow\infty$. Under quadratic approximation of $G(\alpha\,;\,s)$, we have*

(a) The asymptotic maximum expected logarithmic utility from terminal wealth for the ith investor, with initial capital $x>0$, is

$$u_T^i(x)\approx\log x+\frac{1}{2}\int_0^T\left(\frac{\mu_t}{\sigma_t}\right)^2dt+\int_0^T Q\left(\frac{\mu_t}{\sigma_t}\right)dt +\frac{\tilde{\lambda}}{4}(1-p)(1+(-1)^{i+1}p)\left[1+\int_{\mathbf{R}}z^2\,\nu_U(dz)\right]T.$$

(b) *The asymptotic excess expected logarithmic utility of the informed investor is*

$$u_T^1(x) - u_T^0(x) \approx \frac{\tilde{\lambda}}{2}\, p\,(1-p)\left[1 + \int_{\mathbf{R}} z^2\, \nu_U(dz)\right] T,$$

where $\tilde{\lambda} = \lambda\gamma^2$ *is the long-run adjusted mean-reversion speed given.*

(c) *Let* λ *be the speed of the original mispricing process. There exists an adjusted continuous O–U mispricing process with speed* $\hat{\lambda} = \frac{[1+\int_{\mathbf{R}} z^2\, \nu_U(dz)]\sigma_\infty^2}{\sigma_\infty^2 + M(2\,;\,q\sigma_\infty)}$, *and a continuous diffusion process linked to the original asset price process with volatility* σ_t, *such that the excess optimal expected utility of the informed investor is*

$$u_T^1(x) - u_T^0(x) \approx \frac{\hat{\lambda}}{2}\, p\,(1-p)T.$$

Moreover, the excess utility is greater (less) than its continuous geometric Brownian motion (GBM) counterpart if the product of σ_∞^2 *and the quadratic variation* $\int_{\mathbf{R}} z^2\, \nu_U(dz)$ *of the jump component of U is greater (less) than* $M(2\,;\,q\sigma_\infty)$, *the second instantaneous centralized moment of return of the asset price jump process in the long-run.*

Observe from Part (a) that under quadratic approximation, the long-run optimal expected utility of each investor is the sum of four components. The first two components are continuous, while the last two components depend on the jumps in both asset price and mispricing processes through their respective Lévy measures. The level of mispricing q, its adjusted speed $\tilde{\lambda}$, diffusive volatility σ_t, and investment period T are also important ingredients that influence optimal expected utility.

Part (b) presents a version of the excess optimal expected utility of the informed investor over the uninformed that depends on the adjusted mean-reversion speed $\tilde{\lambda}$ of the mispricing process, which has been dampened by the total variance of the asset price process. It also depends on the level of mispricing and the Lévy measure governing the mispricing process. Our model shows that the informed investor has more long-run utility, and therefore, it pays to be more informed in the long-run, unless there is too little ($q \approx 0$) or too much ($q \approx 1$) mispricing in the market. Excess optimal expected utility is maximized when the percentage of mispricing reaches 75%.

Part (c) is another representation of the long-run excess optimal expected utility that is identical in structure to Buckley et al. (2014) and Guasoni (2006), and indeed nests the mean reversion speeds presented therein. In this case, the model is equivalent to a continuous diffusion with a continuous O–U mispricing process having a new adjusted mean-reversion speed $\hat{\lambda}$, that is a function of the original speed λ, the diffusive volatility, and variances which are constructed from the Lévy measures of both asset price and mispricing jump processes. Observe that if there are no jumps, the model reduces to the purely continuous model of Guasoni (2006).

In our more complete framework, the excess utility can be more or less than its continuous GBM counterpart and is entirely governed by the jump intensities. In particular, the excess optimal expected utility is greater (less) than its continuous GBM counterpart if the product of σ_∞^2 and the quadratic variation $\int_{\mathbf{R}} z^2\, \nu_U(dz)$ of the jump component of U is greater (less) than $M(2\,;\,q\sigma_\infty)$, the second instantaneous centralized moment of return of the asset price jump process in the long-run. In the case where there are no jumps in mispricing, the quadratic variation due to mispricing is zero and consequently the excess utility is always less than the continuous GBM benchmark. As in prior studies by Guasoni (2006) and Buckley et al. (2012, 2014), we obtain maximum excess optimal expected utility when mispricing in the market is at 75%. It is worth noting that whether or not jumps exist, there is no utility advantage for the informed investor if the market is totally free of mispricing or completely mispriced.

Our model could be applied to options pricing that incorporates asymmetric information and mispricing of the underlying asset. It could be further generalized to a multi-market setting with at least two correlated risky assets, and their respective mispricings processes. Preferences for investors could be general CRRA type.

5. Conclusion

We extend the theory of mispricing models under asymmetric information and logarithmic preferences to the discontinuous framework where jumps in both the asset price and mispricing process are incorporated. We obtain analogous but more general results than those presented in Buckley et al. (2014) and Guasoni (2006). We derive explicit formulas for the optimal portfolios and maximum expected logarithmic utilities for both investors under reasonable assumptions. In particular, we show that the optimal portfolio of each investor contains excess stock holdings which depend on the Lévy measure of the jump processes driving both the stock price and mispricing processes, through their instantaneous centralized moments of return.

Under quadratic approximation of the optimal portfolios, we find that the optimal expected utility from terminal wealth for each investor is increased by the presence of jumps in the mispricing. Thus, investors are better off when mispricing jumps than when it changes continuously. We also show that the uninformed investor has more utility resulting from jumps in mispricing. However, the informed investor still has positive excess utility over the uninformed, which suggests that the informed investor gets more utility from the diffusive component driving the asset price process. Furthermore, we show that the asymptotic excess maximum expected utility from terminal wealth of the informed investor is $\frac{\tilde{\lambda}}{2}\, p\,(1-p)\left[1 + \int_{\mathbf{R}} x^2\, \nu_U(dx)\right] T$, which is similar to that presented in Buckley et al. (2014) and Guasoni (2006), but increased by a factor directly attributed to the jumps in mispricing and asset price. As in the case of jumps in asset price only, the mean-reversion speed λ is replaced by an adjusted mean-reversion speed $\tilde{\lambda}$, which is a scalar multiple of the original speed used in the Brownian motion case and dampened by the total volatility of the stock including the component produced by the jumps. Equivalently, the excess optimal expected utility is $\frac{\hat{\lambda}}{2}\, p\,(1-p)T$, where $\hat{\lambda}$ is the mean-reversion speed of an adjusted continuous O–U mispricing process which depends on the volatility of the long-run asset price. In this framework, the excess optimal utility of the informed investor is greater (less) than its continuous Merton (1971) GBM counterpart if the product of σ_T^2 and the quadratic variation $\int_{\mathbf{R}} z^2\, \nu_U(dz)$ of the jump component of U is greater (less) than $M(2\,;\,q\sigma_T)$, the second instantaneous centralized moment of return of the asset price jump process in the long-run.

We also show that the uninformed investor has more expected utility resulting from jumps in mispricing in the long-run, which is larger than what obtained in the Brownian motion case. Despite this counter-intuitive and surprising result, the informed investor still has positive excess utility over the uninformed, which clearly suggests that the informed investor gets more utility from the diffusive component of the asset price process. Thus, notwithstanding the presence of asymmetric information, mispricing and jumps, our results show that it still pays to be more informed in the long-run, unless there is too little or too much mispricing in the market. In this case, the informed investor has no utility advantage.

Our study can be extended as follows. First, it could be applied to specific asset classes such as currencies or indices. Second, our model assumes that volatility is constant or deterministic. It is well-established empirically that volatility is stochastic, and hence, it would be natural to extend the model to the stochastic volatility case. Third, it could also be generalized to multiple risky assets under constant or stochastic volatility, where investors have CRRA preferences. Fourth, the pricing of options under asymmetric information and mispricing has not received much attention in the literature. This could be another fruitful direction for future research.

Appendix A. Proofs

A.1. Proof of Proposition 1

Proof. By the martingale property of Z, $\mathbf{E}(U_t^Z) = \mathbf{E}\int_0^t e^{-\lambda(t-s)}dZ_s = 0$. By Itô-isometry, and the fact that $t\int_{\mathbf{R}} z^2\,\nu_U(dz) = \mathbf{E}(Z_t^2) < \infty$, we have

$$\begin{aligned}\mathbf{Var}(U_t^Z) &= \mathbf{E}(U_t^Z)^2 = \mathbf{E}\left(\int_0^t e^{-\lambda(t-s)}dZ_s\right)^2\\ &= \mathbf{E}\int_0^t e^{-2\lambda(t-s)}d[Z,Z]_s\\ &= \int_0^t e^{-2\lambda(t-s)}\int_{\mathbf{R}} z^2\,\nu_U(dz)ds\\ &= \int_0^t e^{-2\lambda(t-s)}ds\int_{\mathbf{R}} z^2\,\nu_U(dx)\\ &= \frac{1-e^{-2\lambda t}}{2\lambda}\int_{\mathbf{R}} z^2\,\nu_U(dz).\end{aligned}$$

The last result follows directly by letting $t\to\infty$. □

A.2. Proof of Proposition 2

Proof. By Eq. (8), $U_t \triangleq U_t^B + U_t^Z$, whence $\mathbf{E}(U_t) = \mathbf{E}(U_t^B) + \mathbf{E}(U_t^Z) = 0$. Since U^B and U^Z are independent processes, then it follows from Proposition 1 that

$$\begin{aligned}\mathbf{E}(U_t^2) &= \mathbf{Var}(U_t) = \mathbf{Var}(U_t^B) + \mathbf{Var}(U_t^Z)\\ &= \frac{1-e^{-2\lambda t}}{2\lambda} + \left(\frac{1-e^{-2\lambda t}}{2\lambda}\right)\int_{\mathbf{R}} z^2\,\nu_U(dz)\\ &= \left(\frac{1-e^{-2\lambda t}}{2\lambda}\right)\left(1+\int_{\mathbf{R}} z^2\,\nu_U(dz)\right).\end{aligned}$$

The last result follows directly by letting $t\to\infty$. □

A.3. Proof of Lemma 1

Proof. Since $Y_t^c \triangleq Y_t - q\,U_t^Z = p\,W_t + q\,U_t^B$ is a continuous Gaussian process (being the sum of two such independent processes), then by Hitsuda (1968) there exists an $\mathcal{F}^0$-Brownian motion B^0 and a process $\phi_t^{0,B}$ adapted to $\mathcal{F}_t^0$, such that $B_t^0 = Y_t^c + \int_0^t \phi_s^{0,B}ds$, where $\phi_s^{0,B} = -\upsilon_s^{0,B} = \lambda\int_0^t e^{-\lambda(t-s)}(1+\gamma(s))dB_s^0$. Since $\mathcal{K}_t^0 = \mathcal{F}_t^0 \vee \sigma(Z_u, X_u : u \le t)$ then $\mathcal{F}_t^0 \subset \mathcal{K}_t^0$, and since B_t^0 and $\phi_t^{0,B}$ are $\mathcal{F}_t^0$-adapted, they are $\mathcal{K}_t^0$-adapted. Thus, $B_t^0 = Y_t - q\,U_t^Z + \int_0^t \phi_s^{0,B}ds = Y_t - q\,U_t^Z - \int_0^t \upsilon_s^{0,B}ds$. □

A.4. Proof of Proposition 4

Proof. We prove the case for $i=1$. From Eq. (12),

$$\mathbf{E}(\upsilon_t^1) = \mathbf{E}(\upsilon_t^{1,B}) + \mathbf{E}(\upsilon_t^Z) = -\lambda\,q\mathbf{E}(U_t^B) - \lambda\,q\mathbf{E}(U_t^Z) = 0.$$

Since U^B and U^Z are independent O–U processes, then with $q^2 = 1-p^2$, we have

$$\begin{aligned}\mathbf{Var}(\upsilon_t^1) &= \mathbf{Var}(\upsilon_t^{1,B}) + \mathbf{Var}(\upsilon_t^Z) = \lambda^2 q^2\,\mathbf{Var}(U_t^B) + \lambda^2 q^2\,\mathbf{Var}(U_t^Z)\\ &= \lambda^2 q^2\left(\frac{1-e^{-2\lambda t}}{2\lambda}\right) + \lambda^2 q^2\left(\frac{1-e^{-2\lambda t}}{2\lambda}\right)\int_{\mathbf{R}} z^2\,\nu_U(dz)\\ &= \frac{\lambda}{2}(1-e^{-2\lambda t})\,q^2\left(1+\int_{\mathbf{R}} z^2\,\nu_U(dz)\right)\\ &= \frac{\lambda}{2}(1-e^{-2\lambda t})\,(1-p)(1+(-1)^{1+1}p)\left(1+\int_{\mathbf{R}} z^2\,\nu_U(dz)\right)\\ &= \frac{\lambda}{2}(1-e^{-2\lambda t})\,(1+p)(1+(-1)^{1}p)\left(1+\int_{\mathbf{R}} z^2\,\nu_U(dz)\right).\end{aligned}$$

The proof for $i=0$ follows similarly. Specifically, by Itô isometry,

$$\begin{aligned}\mathbf{Var}(\upsilon_t^0) &= \mathbf{Var}(\upsilon_t^{0,B}) + \mathbf{Var}(\upsilon_t^Z)\\ &= \lambda^2\mathbf{Var}\int_0^t e^{-\lambda(t-s)}(1+\gamma(s))dB_s^0 + \lambda^2 q^2\,\mathbf{Var}(U_t^Z)\\ &= \lambda^2\int_0^t e^{-2\lambda(t-s)}(1+\gamma(s))^2ds\\ &\quad+\lambda^2 q^2\left(\frac{1-e^{-2\lambda t}}{2\lambda}\right)\int_{\mathbf{R}} z^2\,\nu_U(dz)\end{aligned}$$

By Part 3 of Proposition 3, $\mathbf{Var}\upsilon_t^{i,B} = \mathbf{E}[\upsilon_t^{i,B}]^2 \to \frac{\lambda}{2}(1-p)(1+(-1)^{i+1}p)$ as $t\to\infty$ and hence $\mathbf{Var}\upsilon_t^i = \mathbf{E}[\upsilon_t^i]^2 \to \frac{\lambda}{2}(1+p)(1+(-1)^i p)\,(1+\int_{\mathbf{R}} z^2\,\nu_U(dz))$. □

A.5. Proof of Theorem 1

Proof. Let $i=1$. By Eq. (18), the dynamics for the informed investor is equivalently given by

$$d(I(t)) = (\mu_t^{*,1}dt + \sigma_t\,dB_t^1 + \int_{\mathbf{R}} q\sigma_t z\,N_U(dt,dz) + \int_{\mathbf{R}} x\,N_S(dt,dx), \tag{A.1}$$

where $I(t) = \log S_t \Longleftrightarrow S_t = e^{\log S_t} = e^{I(t)}$. Let $f(x) = e^x$. Then $f \in C^2(\mathbf{R})$ and by Itô's formula (cf. Theorem 4.4.7, Applebaum (2004)), we have from (A.1) that

$$\begin{aligned}d(f(I_t)) &= \frac{df}{dx}(I_{t-}) + \frac{1}{2}\frac{d^2f}{dx^2}(I_{t-})d[I^c, I^c]_t\\ &\quad+\int_{\mathbf{R}}(f(I_{t-}+q\,x\,\sigma_t) - f(I_{t-}))\,N_U(dt,dx)\\ &\quad+\int_{\mathbf{R}}(f(I_{t-}+x) - f(I_{t-}))\,N_S(dt,dx)\\ &= e^{I_{t-}}\left[\mu_t^{*,1}\,dt + \sigma_t\,dB_t^1 + \frac{1}{2}\sigma_t^2dt\right]\\ &\quad+\int_{\mathbf{R}} e^{I_{t-}}(e^{q\sigma_t x}-1)N_U(dt,dx)\\ &\quad+\int_{\mathbf{R}} e^{I_{t-}}(e^{x}-1)N_S(dt,dx)\\ &= e^{I_{t-}}\left[\mu_t^1 dt + \sigma_t dB_t^1 + \int_{\mathbf{R}}(e^{q\sigma_t x}-1)N_U(dt,dx)\right.\\ &\quad\left.+\int_{\mathbf{R}}(e^{x}-1)N_S(dt,dx)\right].\end{aligned}$$

Thus, we get

$$\begin{aligned}d(e^{\log S_t}) &= S_{t-}\left[\mu_t^1 dt + \sigma_t dB_t^1 + \int_{\mathbf{R}}(e^{q\sigma_t z}-1)N_U(dt,dz)\right.\\ &\quad\left.+\int_{\mathbf{R}}(e^{x}-1)N_S(dt,dx)\right]\end{aligned}$$

which is equivalent to the percentage return equation:

$$\frac{dS_t}{S_{t-}} = \mu_t^1 dt + \sigma_t dB_t^1 + \int_{\mathbf{R}} (e^{q\sigma_t z} - 1) N_U(dt, dz) + \int_{\mathbf{R}} (e^x - 1) N_S(dt, dx). \tag{A.2}$$

The proof for the uninformed investor is identical. □

A.6. Proof of Theorem 2

Proof. We provide the proof for the informed investor. The crucial component of this proof is Lemma 2, which states that $N_U(t, A)$ and $N_S(t, A)$ do not jump at the same time, because they are independent Poisson processes on each bounded Borel set $A \in \mathcal{B}(\mathbf{R} - \{0\})$. From Theorem 1, the percentage return of V^1 is

$$\begin{aligned}\frac{dV_t^1}{V_{t-}^1} &= (1-\pi_t) r_t dt + \pi_t \frac{dS_t}{S_t} \\ &= (1-\pi_t) r_t\, dt + \pi_t\, \mu_t^1\, dt + \pi_t\, \sigma_t\, dB_t^1 \\ &\quad + \int_{\mathbf{R}} \pi_t (e^{q\sigma_t z} - 1) N_U(dt, dz) + \int_{\mathbf{R}} \pi_t (e^x - 1) N_S(dt, dx). \\ &= (r_t + \pi_t\, \sigma_t\, \theta_t^1) dt + \pi_t\, \sigma_t\, dB_t^1 + \int_{\mathbf{R}} \pi_t (e^{q\sigma_t z} - 1) N_U(dt, dz) \\ &\quad + \int_{\mathbf{R}} \pi_t (e^x - 1) N_S(dt, dx).\end{aligned}$$

This proves Eq. (23). By applying the result on Doleans–Dade/Stochastic exponentials, and using the fact of the independence of jumps of N_U and N_S, Eq. (23) yields the solution

$$V_t^1 = V_0^1 \exp\left(\int_0^t r_s ds + \int_0^t \left(\pi_s \sigma_s \theta_s^1 - \frac{1}{2} \pi_s^2 \sigma_s^2 \right) ds + \int_0^t \pi_s \sigma_s dB_s^1 \right) \times \Pi_{0 \le u \le t} (1 + \pi_u (e^{q\sigma_u \Delta Z_u} - 1)) \times \Pi_{0 \le s \le t} (1 + \pi_s (e^{\Delta X_s} - 1)),$$

with discounted wealth given by (24). □

A.7. Proof of Lemma 3

Proof. Let $a \in \{U, S\}$ and $s \in I_\sigma$. By the standing assumption, $\int_{\mathbf{R}} (e^{\pm sx} - 1)^2 \nu_a(dx) < \infty$. Thus $G_U, G_S \in C^2[0, 1]$, and for all $\alpha \in [0, 1]$ and $s \in I_\sigma$, $G_a''(\alpha\,;\, s) = -\int_{\mathbf{R}} \frac{(e^{sx}-1)^2 \nu_a(dx)}{(1+\alpha(e^{sx}-1))^2} < 0$. Thus, $G''(\alpha\,;\, s) = G_U''(\alpha\,;\, s) + G_S''(\alpha\,;\, 1) < 0$. □

A.8. Proof of Theorem 3

Proof. We provide the proof for the informed investor ($i = 1$). It is similar for the uninformed investor ($i = 0$). Since utility is logarithmic, then using Poisson integration, the utility from terminal (discounted) wealth for the informed investor is:

$$\begin{aligned}\log \widetilde{V}_T^1 &= \log x + \int_0^T \left(\pi_s \sigma_s \theta_s^1 - \frac{1}{2} \pi_s^2 \sigma_s^2 \right) ds + \int_0^T \pi_s \sigma_s dB_s^1 \\ &\quad + \sum_{0 \le u \le T} \log(1 + \pi_u (e^{q\sigma_u \Delta Z_u} - 1)) \\ &\quad + \sum_{0 \le s \le T} \log(1 + \pi_s (e^{\Delta X_s} - 1)) \\ &= \log x + \int_0^T \left(\pi_s \sigma_s \theta_s^1 - \frac{1}{2} \pi_s^2 \sigma_s^2 \right) ds + \int_0^T \pi_s \sigma_s dB_s^1 \\ &\quad + \int_0^T \int_{\mathbf{R}} \log(1 + \pi_u (e^{q\sigma_u z} - 1)) N_U(du, dz) \\ &\quad + \int_0^T \int_{\mathbf{R}} \log(1 + \pi_s (e^x - 1)) N_S(ds, dx)\end{aligned}$$

Taking expectation yields

$$\begin{aligned}\mathbf{E} \log \widetilde{V}_T^1 &= \log x + \frac{1}{2} \mathbf{E} \int_0^T (\theta_s^1)^2 ds - \frac{1}{2} \mathbf{E} \int_0^T (\pi_s \sigma_s - \theta_s^1)^2 ds \\ &\quad + \int_0^T \int_{\mathbf{R}} \log(1 + \pi_u (e^{q\sigma_u z} - 1)) \nu_U(dz) ds \\ &\quad + \int_0^T \int_{\mathbf{R}} \log(1 + \pi_s (e^x - 1)) \nu_S(dx) ds\end{aligned}$$

Thus

$$\mathbf{E} \log \widetilde{V}_T^1 = \log x + \frac{1}{2} \mathbf{E} \int_0^T (\theta_t^1)^2 dt + \mathbf{E} \int_0^T f_{U,S}^{(1)}(\pi_t) dt, \tag{A.3}$$

where

$$\begin{aligned}f_{U,S}^{(1)}(\pi_t) &= -\frac{1}{2} (\pi_t \sigma_t - \theta_t^1)^2 + \int_{\mathbf{R}} \log(1 + \pi_t (e^{q\sigma_t z} - 1)) \nu_U(dz) \\ &\quad + \int_{\mathbf{R}} \log(1 + \pi_t (e^x - 1)) \nu_S(dx) \\ &= G_U(\pi_t; q\sigma_t) + G_S(\pi_t; 1) - \frac{1}{2} (\pi_t \sigma_t - \theta_t^1)^2 \\ &= G(\pi_t; q\sigma_t) - \frac{1}{2} (\pi_t \sigma_t - \theta_t^1)^2.\end{aligned}$$

The value function is

$$\begin{aligned}u^1(x) &= \max_{\pi \in \mathcal{A}_1(x)} \mathbf{E} \log \widetilde{V}_T^{1,\pi} = \log x + \frac{1}{2} \mathbf{E} \int_0^T (\theta_t^1)^2 dt \\ &\quad + \max_\pi \mathbf{E} \int_0^T f_{U,S}^{(1)}(\pi_s) ds \\ &= \log x + \frac{1}{2} \mathbf{E} \int_0^T (\theta_t^1)^2 dt + \mathbf{E} \int_0^T \max_\pi f_{U,S}^{(1)}(\pi_s) ds.\end{aligned} \tag{A.4}$$

Thus, the objective function $f_{U,S}^{(1)}(\pi)$ given by (31) is strictly concave by Lemma 3 if $\pi \in [0, 1]$, $\int_{\mathbf{R}} (e^{\pm\sigma x} - 1)^2 \nu_a(dx) < \infty$ and $\int_{\mathbf{R}} (e^{\pm x} - 1)^2 \nu_a(dx) < \infty$, where $a \in \{U, S\}$. Thus, $f_{U,S}^{(1)}(\pi)$ has a maximum at π^1 where $\frac{d}{d\pi} (f_{U,S}^{(1)}(\pi))|_{\pi = \pi^1} = 0$. Dropping the subscripts "U" and "S", we have

$$\begin{aligned}f'(\pi) &= G'(\pi\,;\, q\sigma) - \sigma\,(\pi\sigma - \theta^1) \\ &= G_U'(\pi\,;\, q\sigma) + G_S'(\pi\,;\, 1) - \sigma\,(\pi\sigma - \theta^1) = 0,\end{aligned}$$

whence Eq. (27) holds. The maximum expected utility $u^1(x)$ follows from the value function (A.4) with $\max_\pi f_{U,S}^{(1)}(\pi) = f_{U,S}^{(1)}(\pi^1)$. □

A.9. Proof of Lemma 4

Proof. Obviously, $K_a(j\,;\, s) = \int_{\mathbf{R}} (e^{jsx} - 1)\, \nu_a(dx) = K_a(js)$. Thus $K(j\,;\, s) = K_U(j\,;\, s) + K_S(j\,; 1) = K_U(js) + K_S(j)$, whence

$$\begin{aligned}M(k\,;\, s) &= \sum_{j=1}^k (-1)^{j-1} \binom{k}{j} K(j\,;\, s) \\ &= \sum_{j=1}^k (-1)^{j-1} \binom{k}{j} (K_U(j\,;\, s) + K_S(j\,;\, 1)) \\ &= \sum_{j=1}^k (-1)^{j-1} \binom{k}{j} K_U(j\,;\, s) + \sum_{j=1}^k (-1)^{j-1} \binom{k}{j} K_S(j\,;\, 1) \\ &= M_U(k\,;\, s) + M_S(k\,;\, 1).\end{aligned}$$

□

A.10. Proof of Theorem 5

Proof. Part (a): Assume that $\int_{\mathbf{R}} (e^{\pm sx} - 1)^2 \nu(dx) < \infty$, where $s \in I_\sigma$ and G is restricted to $[0, 1]$. By Proposition 4, as $t \to \infty$

$$\mathbf{E}(\upsilon_t^i)^2 \longrightarrow \frac{\lambda}{2} (1-p)(1 + (-1)^{i+1} p) \left[1 + \int_{\mathbf{R}} z^2\, \nu_U(dz) \right], \quad i \in \{0, 1\}.$$

Similar to the proof of Theorem 3 in Buckley et al. (2014), as $T \to \infty$, we have

$$\begin{aligned} u^i_{T,d}(x) &\sim \mathbf{E}\int_0^T Q\left(\frac{\mu_t}{\sigma_t} : \sigma_t,\ M(1; q\sigma_t),\ M(2; q\sigma_t)\right) dt + \int_0^T A_t \mathbf{E}(\upsilon^i_t)^2 dt \\ &\sim \mathbf{E}\int_0^T Q\left(\frac{\mu_t}{\sigma_t}\right) dt + T \lim_{t\to\infty} A_t \mathbf{E}(\upsilon^i_t)^2 \\ &= \mathbf{E}\int_0^T Q\left(\frac{\mu_t}{\sigma_t}\right) dt + T A_\infty \lim_{t\to\infty} \mathbf{E}(\upsilon^i_t)^2 \\ &= \mathbf{E}\int_0^T Q\left(\frac{\mu_t}{\sigma_t}\right) dt \\ &\quad + \frac{\lambda}{2} A_\infty (1-p)(1+(-1)^{i+1}p)\left[1+\int_{\mathbf{R}} z^2\, \nu_U(dz)\right] T, \end{aligned}$$

where $A_\infty = \lim_{t\to\infty} A_t = \lim_{t\to\infty} -\frac{M(2;q\sigma_t)}{2(\sigma_t^2 + M(2;q\sigma_t))} = -\frac{M(2;q\sigma_\infty)}{2(\sigma_\infty^2 + M(2;q\sigma_\infty))}$.

Part (b): From part(a), since $\frac{\lambda}{2} A_\infty \int_{\mathbf{R}} z^2\, \nu_U(dz) T$ is common for both investors, it follows that the excess optimal utility of the uninformed investor due to the jumps is

$$\begin{aligned} u^0_{T,d}(x) - u^1_{T,d}(x) &\approx -\frac{\lambda}{2} A_\infty [(1-p)(1+p) - (1-p)(1-p)] \\ &\quad \times \left[1+\int_{\mathbf{R}} z^2\, \nu_U(dx)\right] T \\ &= -\lambda A_\infty\, p\,(1-p)\left[1+\int_{\mathbf{R}} z^2\, \nu_U(dz)\right] T, \end{aligned}$$

which is non-negative. □

A.11. Proof of Theorem 6

Proof. (a) The total optimal asymptotic utility of the ith investor is $u^i_T(x) = u^i_{T,c}(x) + u^i_{T,d}(x)$, where the continuous component is

$$u^i_{T,c}(x) \sim \log x + \frac{1}{2}\int_0^T \left(\frac{\mu_t}{\sigma_t}\right)^2 dt + \frac{\lambda T}{4}(1-p)(1+(-1)^{i+1}p) \times \left(1+\int_{\mathbf{R}} z^2\, \nu_U(dz)\right),$$

and the discontinuous component is given by Theorem 5 as

$$u^i_{T,d}(x) \sim \int_0^T Q\left(\frac{\mu_t}{\sigma_t}\right) dt + \frac{\lambda}{2} A_\infty (1-p)(1+(-1)^{i+1}p) \times \left(1+\int_{\mathbf{R}} z^2\, \nu_U(dz)\right) T,$$

where $Q(\cdot)$ also depends on σ_t, $M(1; q\sigma_t)$ and $M(2; q\sigma_t)$.

(b) This follows by taking the difference of the optimal utilities for both investors.

(c) This follows immediately from (b) by replacing $\tilde{\lambda}[1+\int_{\mathbf{R}} z^2\, \nu_U(dz)]$ with $\hat{\lambda}$. □

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