
MR1930130 (2003j:05071)
Li, Hao [Li, Hao[1]] (F-PARIS11-RI)
On a conjecture of Woodall. (English summary)
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05C38 (05C40)

The author gives a very nice solution for an extension of the Woodall problem, proposed in 1975, asking whether every 2-connected graph of order n with at least $k+n/2$ vertices of degree at least k has a cycle of length at least $2k$, i.e., with $c(G) \geq 2k$.

This conjecture appeared as an improvement of an older result of Dirac, dated 1952, that every 2-connected graph of order n and minimum degree k admits a cycle for which $c(G) \geq \min\{2k, n\}$. For this conjecture, in 1985, Häggkvist and Jackson obtained a partial result, in the case $n \leq 3k-2$. Later, Li showed that for a 2-connected graph of order n and for $k \in \mathbb{Z}$, where $4k-6 \geq n$ and where the number of vertices with degree at least k is at least $k+1/2$, $c(G) \geq \min\{2k, n\}$.

With a stronger condition on the connectivity, Häggkvist and Li proved, for integers $k \geq 25$ and for a 3-connected graph G of order n with at least $k+n/2$ vertices of degree at least k, that $c(G) \geq \min\{2k, n\}$. Looking to shorten the proof of this anterior result, Li found the following theorem: If G is a 2-connected graph of order n, with at least $k+n/2$ vertices of degree at least k, then $c(G) \geq \min\{2k-13, n\}$. Trying to prove it, Li uses two lemmas based on the concept of (k, B) connectivity, where $B \subset V(G)$, $k \in \mathbb{Z}$, which is not essentially new because its origins are in [W. T. Tutte, *Connectivity in graphs*, Univ. Toronto Press, Toronto, Ont., 1966; MR **35** #1503] and equally in [N. Deo, *Graph theory with applications to engineering and computer science*, Prentice Hall, Englewood Cliffs, N.J., 1974; MR **50** #12772].

Finally, I must remark on the clarity of the paper's exposition and the strong concatenation of the notions in the proofs, which are very difficult in their technicality. I would say that this paper is the most beautiful that has appeared in the last 3 years in the Journal of Combinatorial Theory, Series B. It could in fact be used as a writing model for other mathematicians.

Reviewed by *Laurenţiu Modan*

[References]

1. J. A. Bondy and U. S. R. Murty, ”Graph Theory with Applications,” Macmillan Press, New York, 1976. MR0411988 (54 #117)
2. G. A. Dirac, Some theorems on abstract graphs, *Proc. London Math. Soc.* (3) **2** (1952), 69–81.
3. R. Häggkvist and B. Jackson, A note on maximal cycles in 2-connected graphs, *Ann. Discrete Math.* **27** (1985), 205–208. MR0821520 (87c:05072)
4. R. Häggkvist and H. Li, Long cycles in graphs with some large degree vertice, preprint.
5. H. Li, ”Woodall's Conjecture on Long Cycles,” Rapport de recherche no 1296, LRI, UMR 8623 CNRS-UPS, Bat. 490, Univ. de Paris sud, 91405-Orsay, France.
6. D. Li and H. Li, ”On Longest Cycles in Graphs,” Rapport de recherche no 1160, LRI, URA 410 du CNRS, Bat. 490, Univ. de Paris sud, 91405-Orsay, France.
7. D. R. Woodall, Maximal circuits of graphs II, *Studia Sci. Math. Hungar.* **10** (1975), 103–109. MR0437395 (55 #10325)