Rigidity of Simplicial Complexes

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A *d-dimensional framework* is a staight line representation of a graph in $\mathbb{R}^d$. It is *rigid* if every continuous motion of the vertices which preserves the lengths of the edges results in a framework which is congruent to the original framework. It is *globally rigid* if every d-dimensional framework which has the same underlying graph and the same edge lengths is congruent to the original framework. It is known that both properties depend only on the underlying graph when the framework is *generic* i.e. the set of coordinates of the vertices is algebraically independent over $\mathbb{Q}$.
After giving a brief introduction to the rigidity of frameworks and the associated *rigidity matroid*, I will survey results on the generic rigidity of the 1-skeletons of simplicial $(d-1)$-complexes in $\mathbb{R}^d$ and then describe a recent result with James Cruickshank and Shin-Ichi Tanigawa which characterises generic global rigidity for triangulated $(d-1)$-manifolds in $\mathbb{R}^d$. More precisely, we show that, if G is the 1-skeleton of a triangulation of a $(d-1)$-manifold M, then G is generically globally rigid in $\mathbb{R}^d$ if and only if G is $(d+1)$-connected and, when $d=2$, M is not homeomorphic to the 2-sphere.

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