3-Polychromatic quadrangulations on surfaces

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A map on a surface means a fixed embedding of a graph on the surface. A quadrangulation is a map with each face quadrilateral. A polychromatic k-coloring of a map $G$ is a color-assignment $c : V(G) \rightarrow \{1, \ldots, k\}$ such that each face has all $k$ colors on its boundary vertices, where $c$ is not necessarily a proper coloring of $G$. The polychromatic number of $G$ is the maximum number $k$ such that $G$ admits a polychromatic $k$-coloring.

In our talk, we would like to deal with polychromatic coloring of quadrangulations. It is known that every quadrangulation on the sphere is 3-polychromatic, but there is a non-4-polychromatic quadrangulation on the sphere. However, focusing on other surfaces, there are many non-3-polychromatic quadrangulations. So, in our talk, we give a sufficient condition for a quadrangulation on a surface to be 3-polychromatic, and in particular, the condition is also necessary for those on the projective plane. Our proof uses a method in knot theory, Reidemeister move, regarding a quadrangulation as a union of several closed curves on surfaces crossing suitably.

This is a joint work with Raiji Mukae (Miyakonojo National College of Technology) and Yusuke Suzuki (Niigata University).

Keywords: quadrangulation, polychromatic coloring, projective plane, Reidemeister move