Diagonal transformations in $N$-angulations on the sphere

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An $N$-angulation is a finite simple plane graph such that each face is bounded by a cycle of length $N$, where $N \geq 3$ is an integer. We consider diagonal transformations in $N$-angulations which consist of $\left\lfloor \frac{N}{2} \right\rfloor$ kinds of operations. In the literature, Wagner proved that any two 3-angulations (which are often called triangulations) with the same number of vertices can be transformed into each other by diagonal transformations. (In this case, a diagonal transformation in 3-angulations is unique, which is called a diagonal flip.) For 4-angulations (which are often called quadrangulations), Nakamoto proved the similar result. Moreover, the similar theorems were recently proved for $N = 5$ and 6. In the paper which includes the result for 6-angulations, the present author conjectured that any two $N$-angulations with the same number of vertices can be transformed into each other by diagonal transformations for any $N \geq 7$. However, it was thought that the proof would be a routine with a case-by-case argument. Then, in this paper, we prove the conjecture by developing a more general technique for proving.

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