Jumps (and non-jumps) in Non-uniform Hypergraphs

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Let $d(H)$ be the edge density of an $r$-uniform hypergraph $H$. A non-negative real number $\alpha$ is a jump for $r$ if there exists a constant $c > 0$ so that for any $\epsilon > 0$ and any $t \geq r$ there exists a $t_0$ such that if $H$ is any $r$-uniform hypergraph satisfying $|H| \geq t_0$ and $d(H) \geq \alpha + \epsilon$ then $H$ contains a subgraph $H'$ on $t$ vertices with $d(H') \geq \alpha + c$. Erdős proved that every $\alpha \in [0, 1)$ was a jump for $r = 2$, i.e. simple graphs, and conjectured that the same was true for all $r$-uniform hypergraphs. In 1984 Frankl and Rödl disproved this conjecture giving the first examples of non-jump values.

In this talk, we generalize the method of Frankl and Rödl yeilding a sufficient condition for $\alpha$ to be a non-jump. One of the big open problems is to find small non-jump values of $\alpha$. The sufficient condition we give, and indeed the method of Frankl and Rödl, is ineffective at finding small non-jump values. By refining the notion of jump we characterize the non-jump values. This characterization provides a method by which one may hope to find small non-jump values.

Keywords: Turán density, hypergraph Langrangian, blowup density, the jumping constant conjecture of Erdős.