On the clique structure of edge intersection graphs of subtrees of a tree

Eli Berger, Elad Cohen*, Irith Ben-Arroyo Hartman, University of Haifa, Israel

The family of *Edge intersection graphs of Paths in a Tree (EPT)* was introduced by Golumbic and Jamison. They showed that the recognition and the coloring problems for this class are NP-complete. However, they presented a polynomial time solution for the maximum clique problem for EPT graphs by characterizing their clique structure. We study the clique structure of *Edge intersection graphs of Subtrees of a Tree (EST)*, a natural generalization of EPT graphs. In particular, we study EST graphs where the maximum degree of the subtrees is bounded, namely $EST_k$ graphs. We introduce a geometrical model, namely *semi-projective planes*, which generalizes the known model of projective planes. We show that the problem of characterizing cliques in $EST_k$ graphs is equivalent to the problem of finding the maximum number of points in a semi-projective plane. We give lower and upper bounds for the maximum number of points in a semi-projective plane of order $k$, for $k \geq 1$, and tight bounds for the cases $k = 1$ and $k = 2$.

Keywords: Intersection graphs, maximum clique, projective plane