Coloring graphs with forbidden induced subgraphs
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Since graph-coloring is an NP-complete problem in general, it is natural to ask how the complexity changes if the input graph is known not to contain a certain induced subgraph \( H \). Due to results of Kaminski and Lozin, and Hoyler, the problem remains NP-complete, unless \( H \) is the disjoint union of paths. Recently the question of coloring graphs with a fixed-length induced path forbidden has received considerable attention.

Only one case of that problem remains open for \( k \)-coloring when \( k \geq 4 \), and that is the case of 4-coloring graphs with no induced 6-vertex path. However, little is known for 3-coloring. Recently we settled the first open case for 3-coloring; namely we showed that 3-coloring graphs with no induced 7-vertex paths can be done in polynomial time. We also made progress on the 4-coloring question. In this talk we will discuss some of the ideas of the algorithms.

This is joint work with Peter Maceli, Juraj Stacho and Mingxian Zhong.