Tree Decompositions of Cayley Graphs and Digraphs with Word-Degenerate Connection Sets

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For a tree $T$, the graph $G$ is $T$-decomposable if there exists a partition of the edge set of $G$ into isomorphic copies of $T$. In 1963, Ringel conjectured that $K_{2m+1}$ can be decomposed by any tree with $m$ edges. In 1989, Häggkvist conjectured more generally that every $2m$-regular graph can be decomposed by any tree with $m$ edges. Fink showed in 1994 that for any directed tree $T$, the directed Cayley graph $\text{Cay}(G; S)$ is $T$-decomposable if $|S| = |E(T)|$ and $S$ is a minimal generating set of $G$. This talk presents an enlarged family of Cayley graphs and digraphs that are tree decomposable. In particular, if our connection set is “$(k, t)$-word degenerate,” and $T$ is a directed tree with minimal spanning star forest $F$, then the directed Cayley graph $\text{Cay}(G; S)$ is $T$-decomposable whenever $|S| = |E(T)|$, $k \geq \text{diam}(T)$, and $t \leq |E(F)|$.

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