On Decomposing Regular Graphs and Multigraphs into Isomorphic Trees

Saad El-Zanati, Illinois State University

Let $H$ and $G$ be graphs or multigraphs such that $G$ is a subgraph of $H$. A $G$-decomposition of $H$ is a set $\Delta = \{G_1, G_2, \ldots, G_t\}$ of pairwise edge-disjoint subgraphs of $H$ each of which is isomorphic to $G$ and such that each edge of $H$ occurs in exactly one $G_i$. Graham and Häggkvist have conjectured that every tree with $n$ edges decomposes every $2n$-regular graph. This conjecture has been confirmed for a small number of cases. If $G$ is a tree with $n$ edges and $H$ is $n$-regular, then $G$ may or may not decompose $H$. For a simple graph $H$, we let $2H$ denote the multigraph obtained by replacing each edge of $H$ with two parallel edges. We have previously conjectured that if a $G$ is a tree with $n$ edges and $H$ is an $n$-regular simple graph, then there exists a $G$-decomposition of $2H$. In this talk, we report on some recent results related to variations of these conjectures.

Keywords: tree decomposition, Graham-Häggkvist Conjecture, 2-fold graph