Edge-Disjoint Hamilton Cycles in Star Graphs of Prime Dimension

Parisa Derakhshan*, Walter Hussak, Loughborough University, UK

The star graphs $S_{tn}$ are Cayley graphs over the symmetric group of permutations with a certain set of generating transpositions. Hamilton decomposition of these graphs have been studied by several authors. Hussak and Schröder found a Hamilton decomposition of $S_{t5}$, and Čada et al. showed that $S_{tn}$ contains $\lfloor n/8 \rfloor$ pairwise edge-disjoint Hamilton cycles where $n$ is prime. The prime integer $n$ is called a safe prime if $n = 2z + 1$ where $z$ is also prime. In this paper, by defining automorphisms and based on the automorphic images of a known Hamilton cycle in $S_{tn}$, we improved this bound to $\lfloor n/4 \rfloor$ where $n$ is a safe prime.

Keywords: Hamilton cycle, star graph, Hamilton decomposition.