**Fractional \((P, Q)\)-Total Colorings of Graphs**

Margit Voigt*, University of Applied Sciences Dresden, Arnfried Kemnitz, Technical University Braunschweig, Anja Pruchnewski, University of Technology Ilmenau

We denote the class of all finite simple graphs by \(I\). A graph property \(P\) is any non-empty isomorphism-closed subclass of \(I\). A property \(P\) of graphs is called hereditary if it is closed under taking subgraphs, i.e., \(G \in P\) and \(H \subseteq G\) implies \(H \in P\). A property \(P\) is called additive if it is closed under disjoint union of graphs, i.e., \(G \in P\) and \(H \in P\) implies \(G \cup H \in P\). Some well-known hereditary and additive graph properties are:

\[
O_k = \{ G \in I : \text{each component of } G \text{ has at most } k+1 \text{ vertices} \}
\]

and

\[
D_k = \{ G \in I : \delta(H) \leq k \text{ for each } H \subseteq G \}, \text{ where } \delta(H) \text{ is the minimum degree of } H.
\]

Let \(r, s \in \mathbb{N}\), \(r \geq s\), and \(P\) and \(Q\) be two additive and hereditary graph properties. A \((P, Q)\)-total independent set \(T = V_T \cup E_T \subseteq V \cup E\) of a graph \(G\) is the union of a set \(V_T\) of vertices and a set \(E_T\) of edges of \(G\) such that for the graphs induced by the sets \(V_T\) and \(E_T\) it holds that \(G[V_T] \in P\), \(G[E_T] \in Q\), and \(G[V_T] \) and \(G[E_T]\) are disjoint. Let \(T_{P,Q}\) be the set of all \((P, Q)\)-total independent sets of \(G\).

A \((P, Q)\)-total \((r, s)\)-coloring of a graph \(G = (V, E)\) is a coloring of all \(x \in V \cup E\) by \(s\)-elements subsets \(C(x) \subseteq \{1, \ldots, r\}\) such that for each color \(i\), \(1 \leq i \leq r\), the set \(T_i := \{ x \in V \cup E : \ i \in C(x) \}\) belongs to \(T_{P,Q}\). The fractional \((P, Q)\)-total chromatic number \(\chi''_{f,P,Q}(G)\) of \(G\) is defined as the infimum of all ratios \(r/s\) such that \(G\) has a \((P, Q)\)-total \((r, s)\)-coloring.

In the talk we give an equivalent definition of the fractional \((P, Q)\)-total chromatic number \(\chi''_{f,P,Q}(G)\) and use that definition to construct bounds for the \((P, Q)\)-total chromatic number for some properties \(P\) and \(Q\). Moreover, we discuss the list coloring version of this concept.

Keywords: total coloring, fractional coloring, hereditary property, list coloring